Optical Metrology

Lecture 2: Random Data and Characterization of Measurement Systems

Content of the Lecture

- Deterministic Data.
- Random Data.
- Characteristics of Random Data.
- Characterization of measurement systems.
- Static and Dynamic characterization.

Deterministic versus Random Data

Deterministic Data

- Any observed data representing a physical phenomenon can be broadly classified as being either deterministic or nondeterministic.
- Deterministic data are those that can be described by an explicit mathematical relationship.

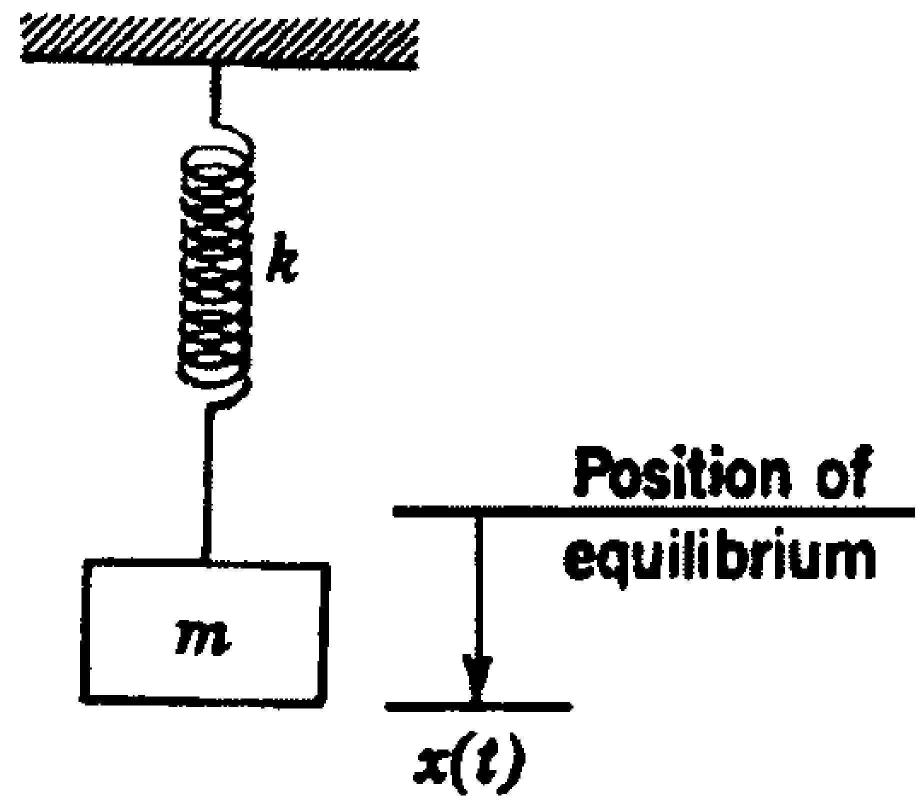
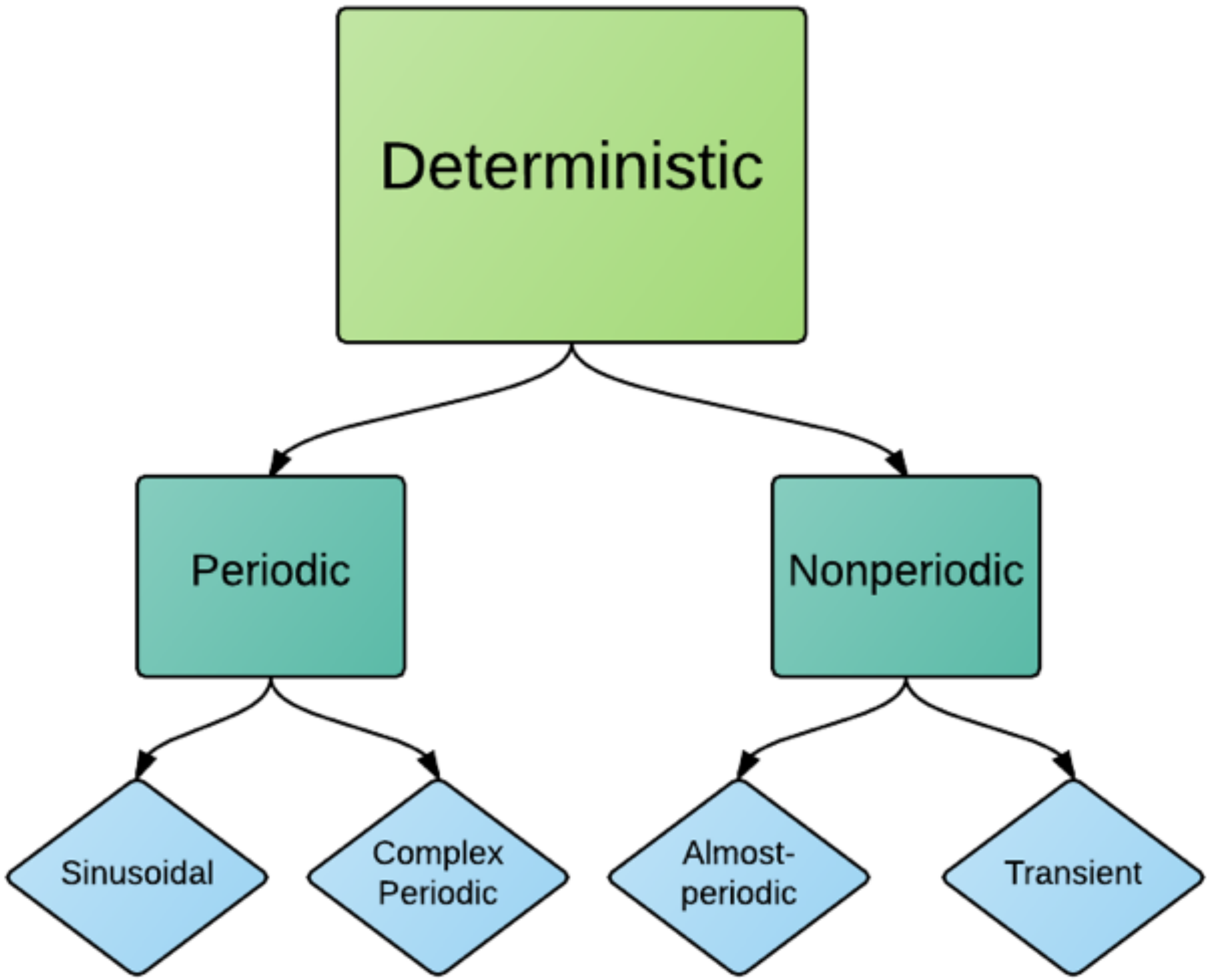


Figure 1.1 Simple spring mass system.

$$x(t) = X \cos \sqrt{\frac{k}{m}} t \quad t \geq 0$$

Classification of Deterministic Data



(Periódico arbitrario)

Sinusoidal

$$x(t) = X \sin(2\pi f_0 t + \phi)$$

Discrete spectra

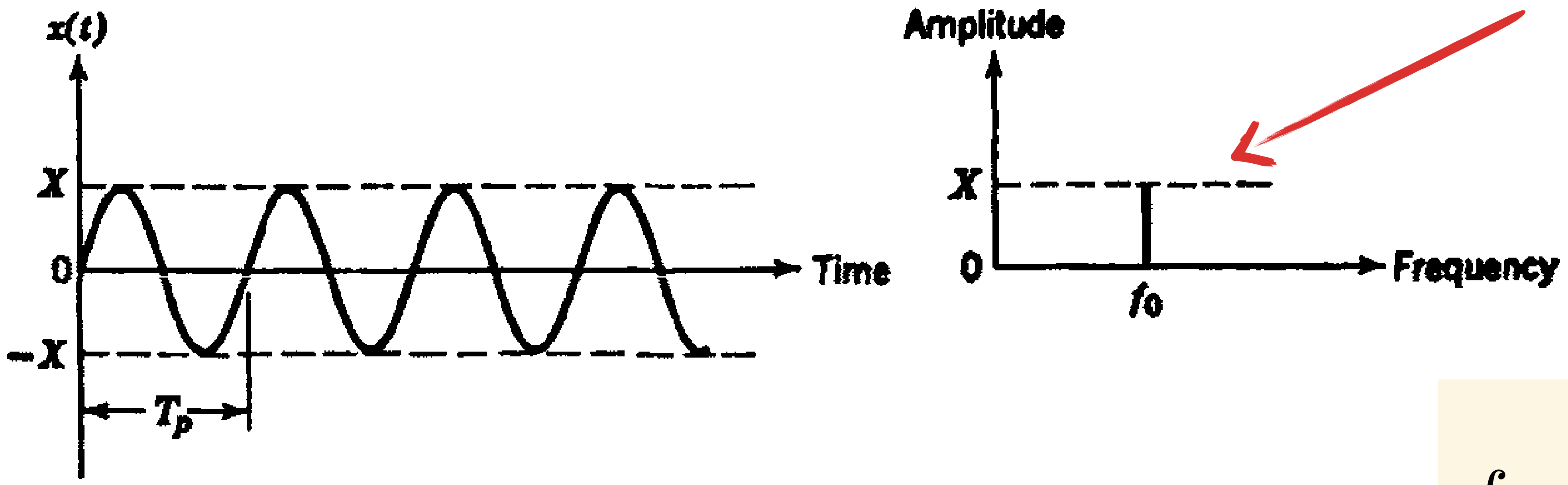


Figure 1.3 Time history and spectrum of sinusoidal data.

$$f_0 = \frac{1}{T_p}$$

Complex Periodic

(Arbitrario)

$$x(t) = X(t \pm nT_p), \quad n = 1, 2, 3, \ldots$$

$$f_1 = \frac{1}{T_p}$$

Data consists of a static component X_0 and an infinite number of sinusoidal components called harmonics. integral multiples of f_1.

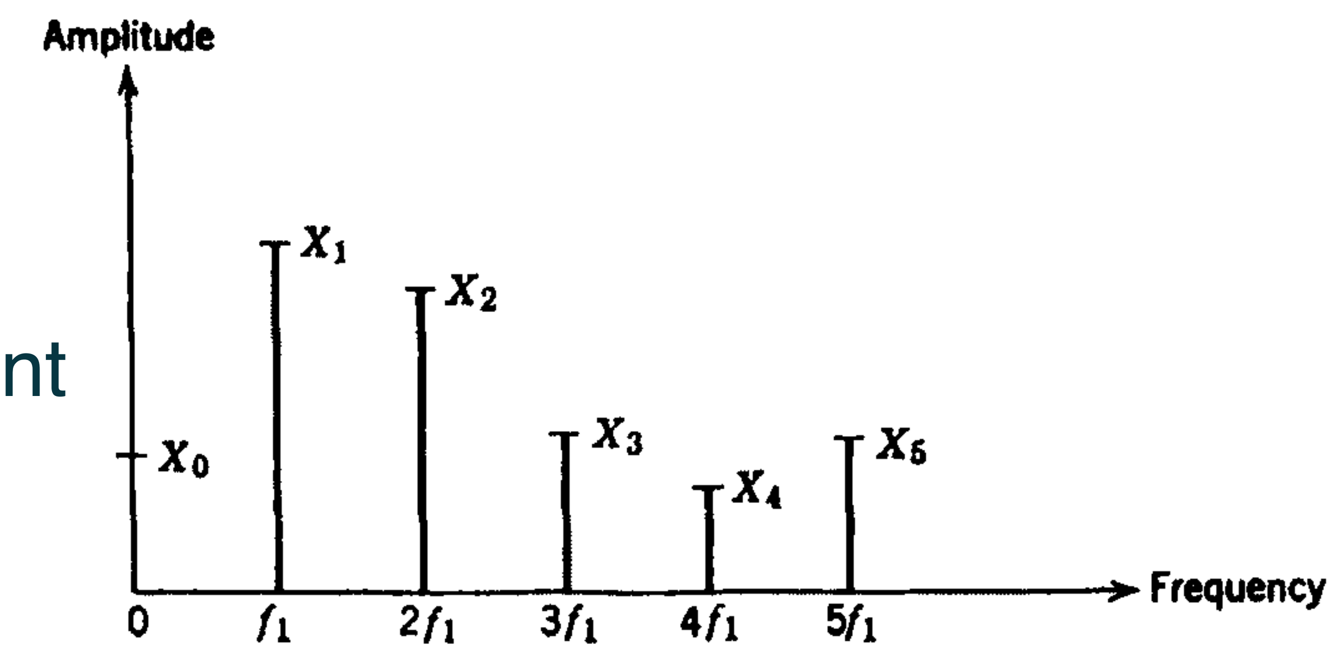


Figure 1.4 Spectrum of complex periodic data.

Almost-periodic

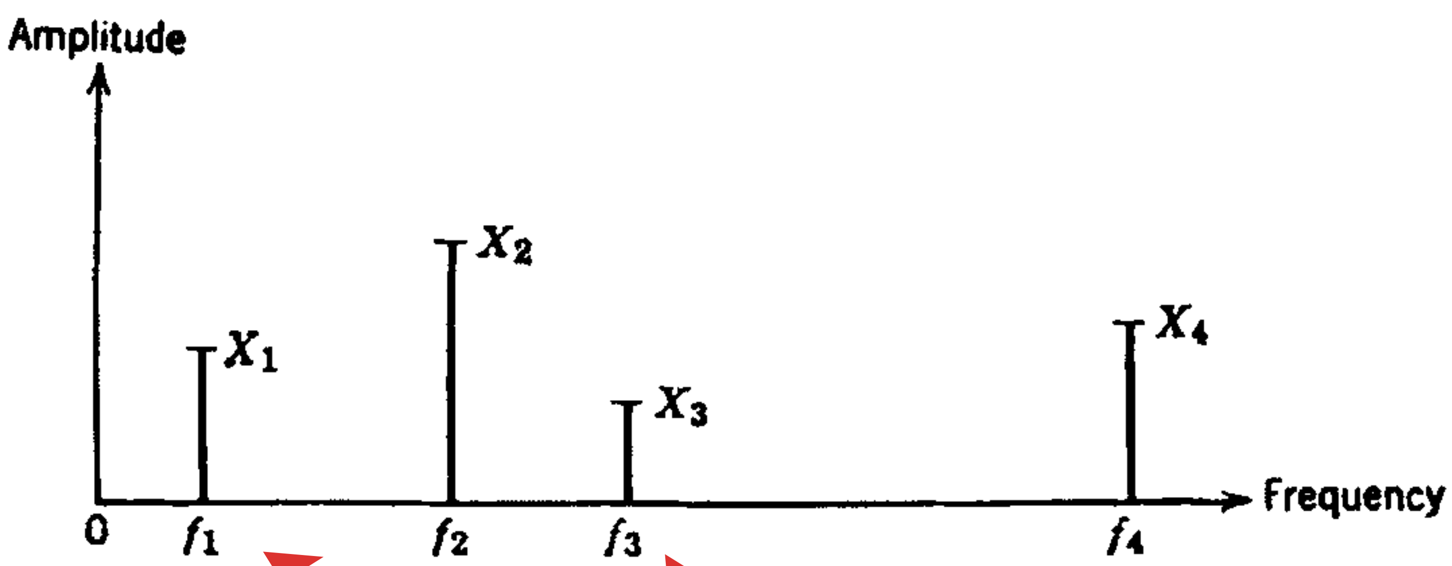


Figure 1.5 Spectrum of almost-periodic data.

No relation

Transient Nonperiodic Data

$$x(t) = \begin{cases} A\, e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(t) = \begin{cases} A\, e^{-at} \cos bt & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(t) = \begin{cases} A & c \geq t \geq 0 \\ 0 & c < t < 0 \end{cases}$$

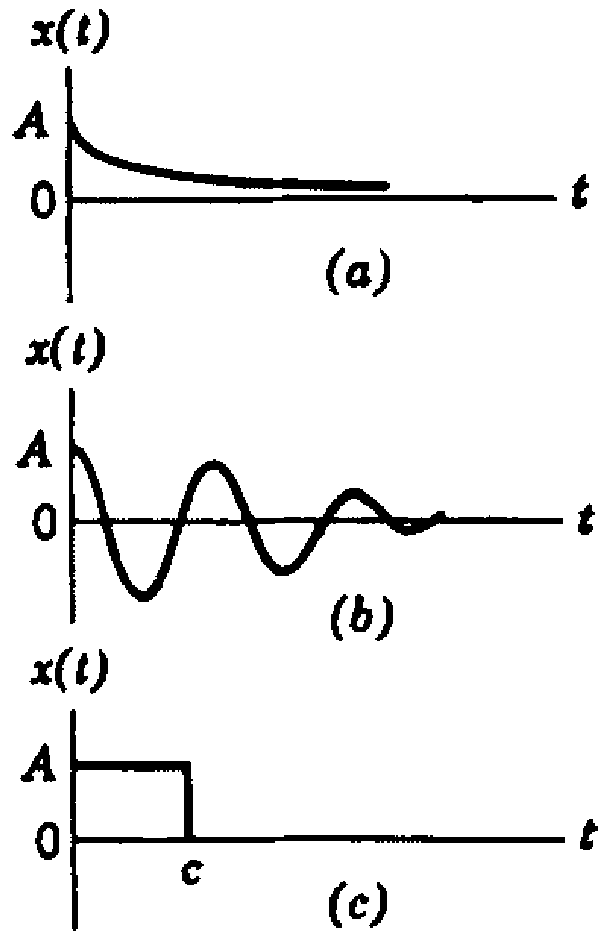


Figure 1.6 Illustrations of transient data.

Continuous spectral representation.

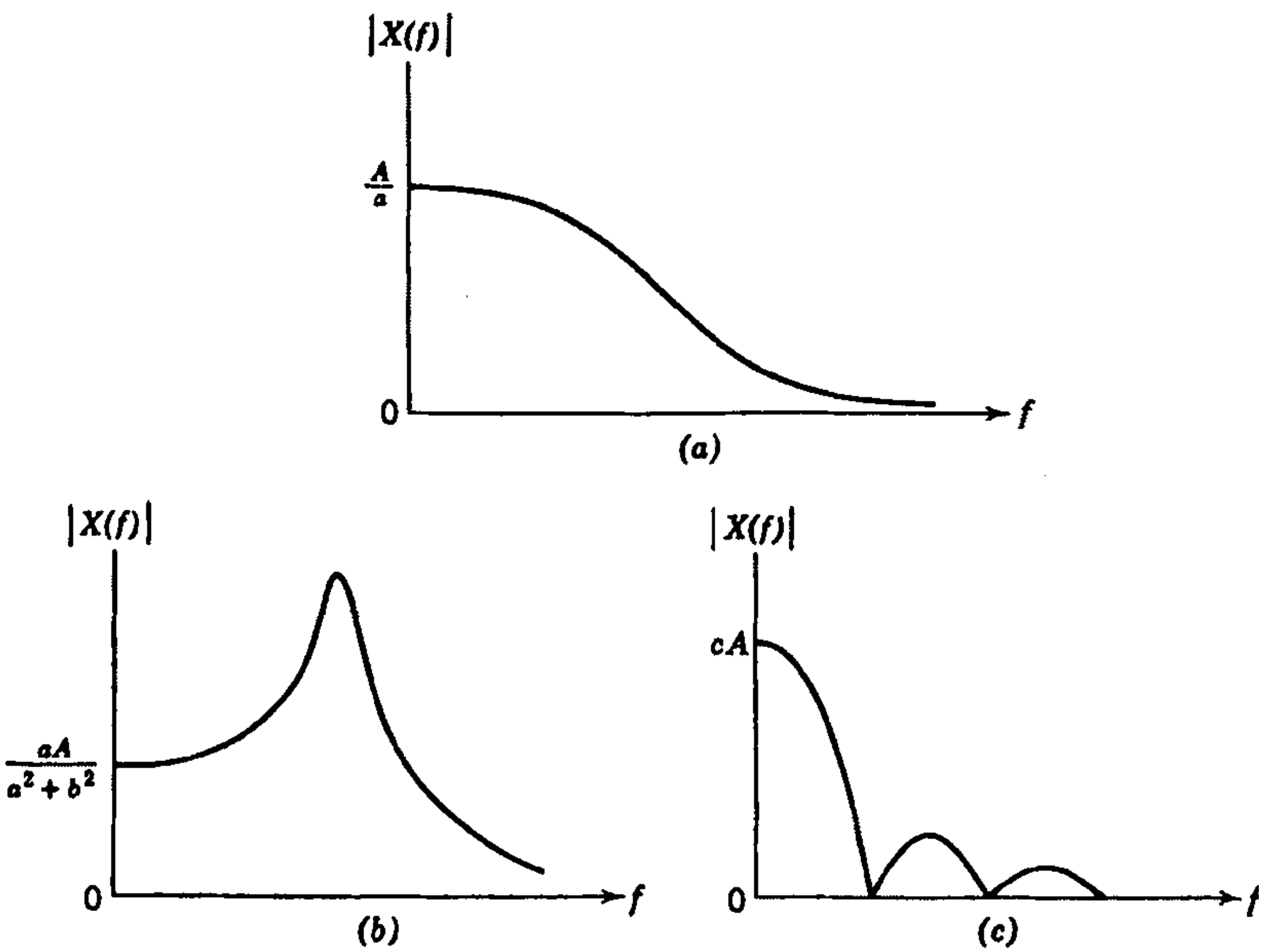
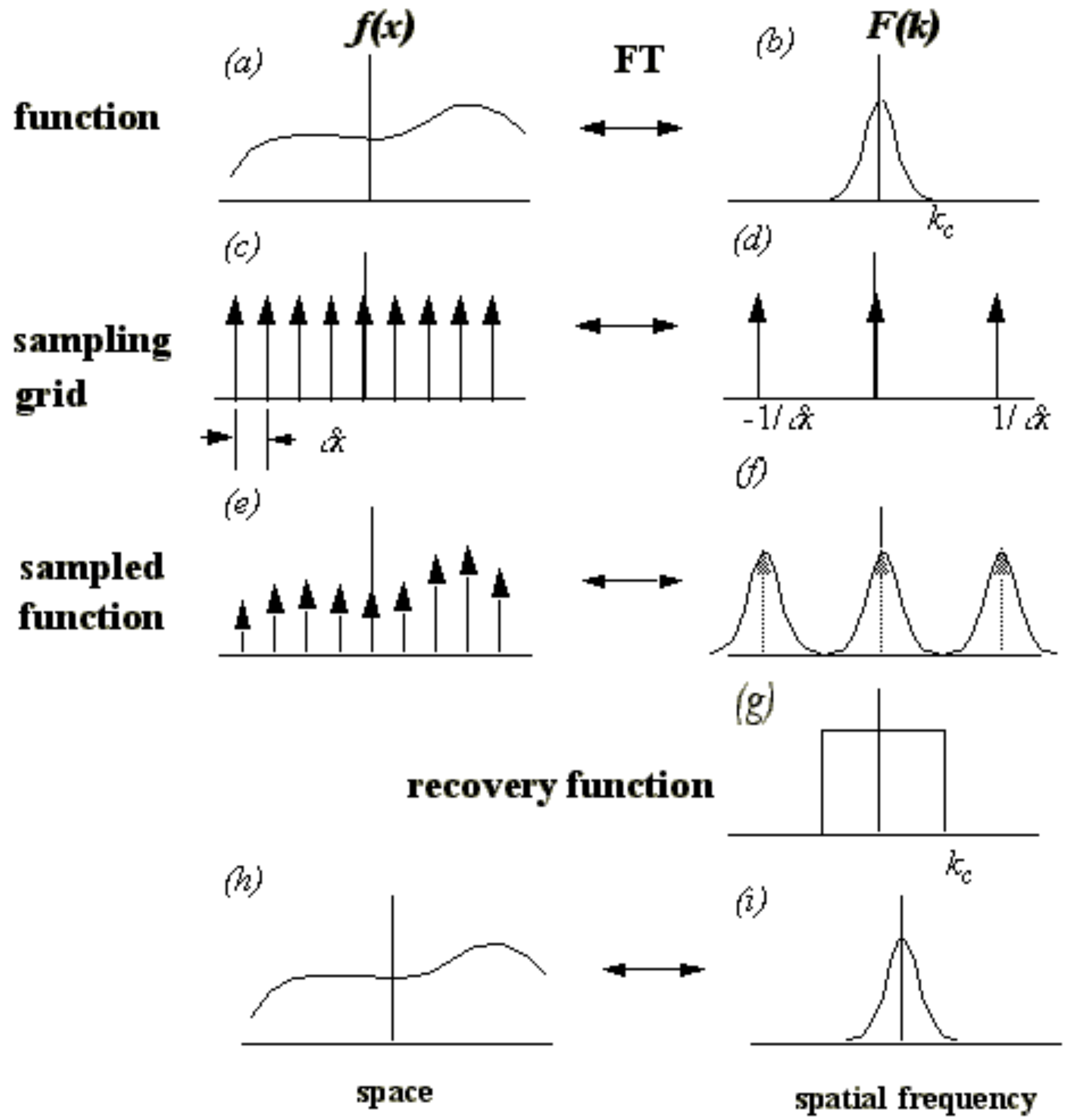
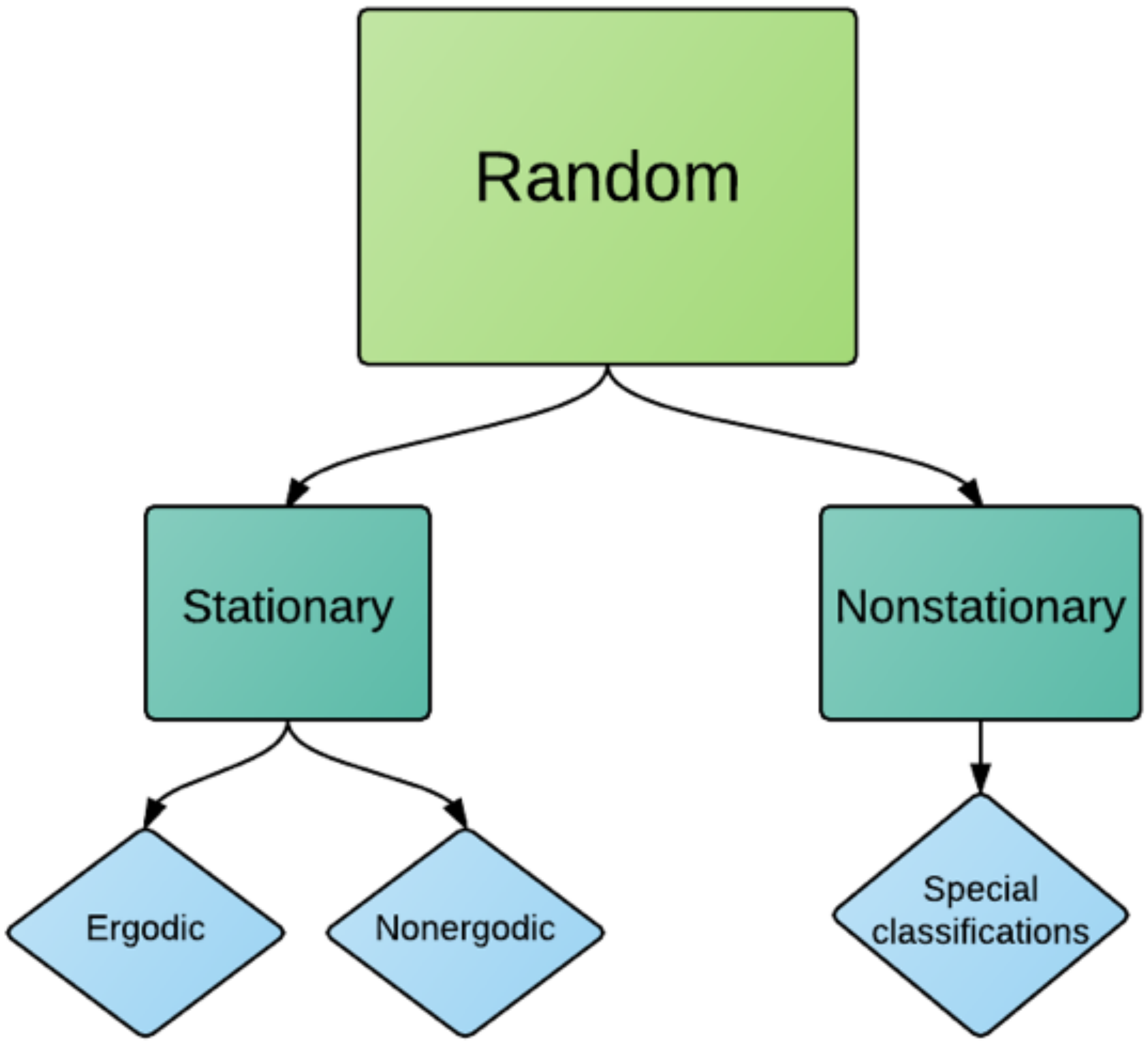


Figure 1.7 Spectra of transient data.

How do you approximate sampling?



Classification of Random Data



Random Data

- A single time history representing a random phenomenon is called a sample function (or a sample record when observed over a finite time interval).

- The collection of all possible sample functions that the random phenomenon might have produced is called a random process or a stochastic process.

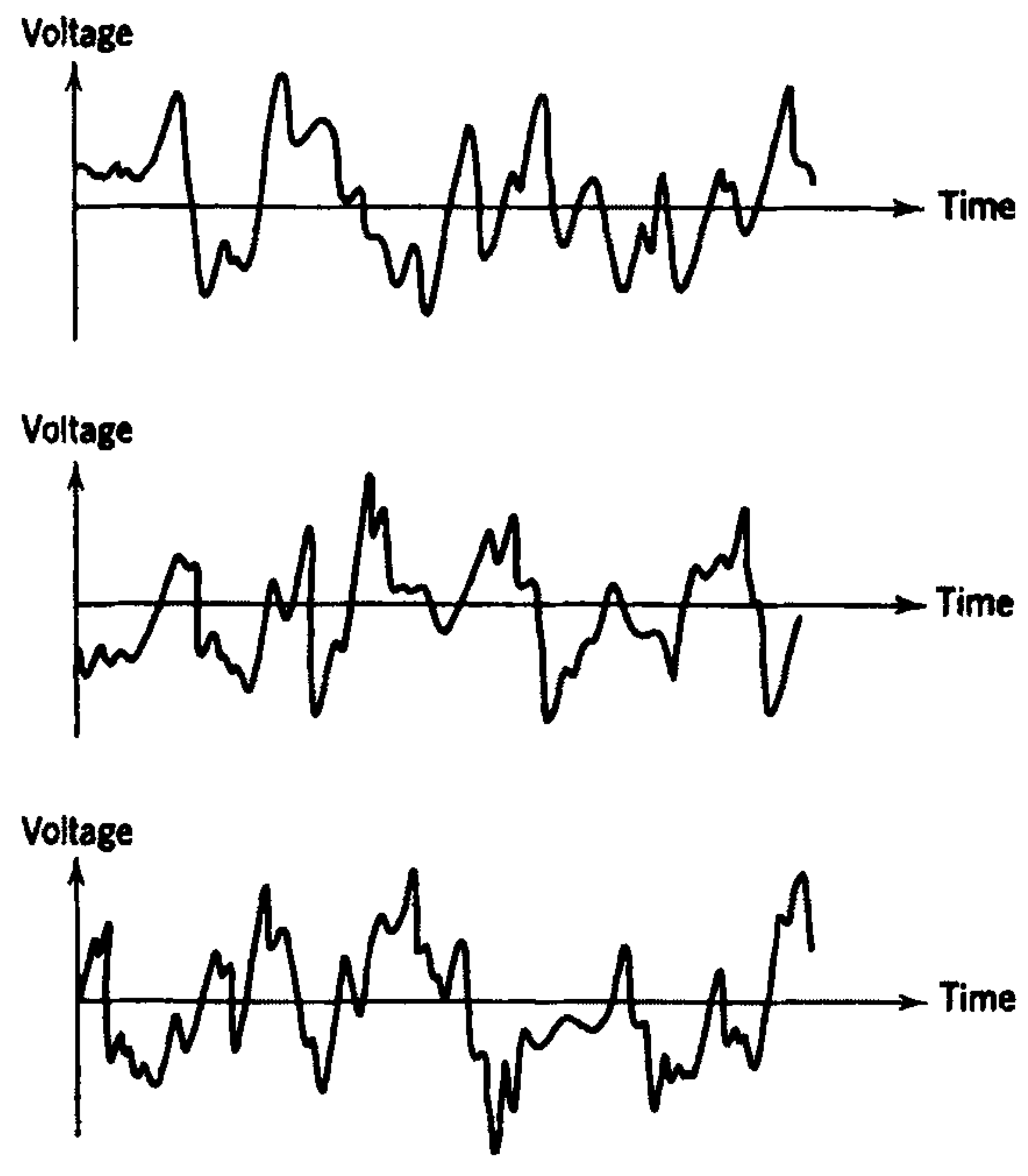


Figure 1.8 Sample records of thermal noise generator outputs.

Stationary Random Data

- A random process can be described by computing average values over the collection of sample functions

$$\mu_x(t_1) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1)$$

$$R_{xx}(t_1, t_1 + \tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1) x_k(t_1 + \tau)$$

- If $\mu_x(t_1)$ and $R_{xx}(t_1, t_1 + \tau)$ vary with t_1, the process is non-stationary.

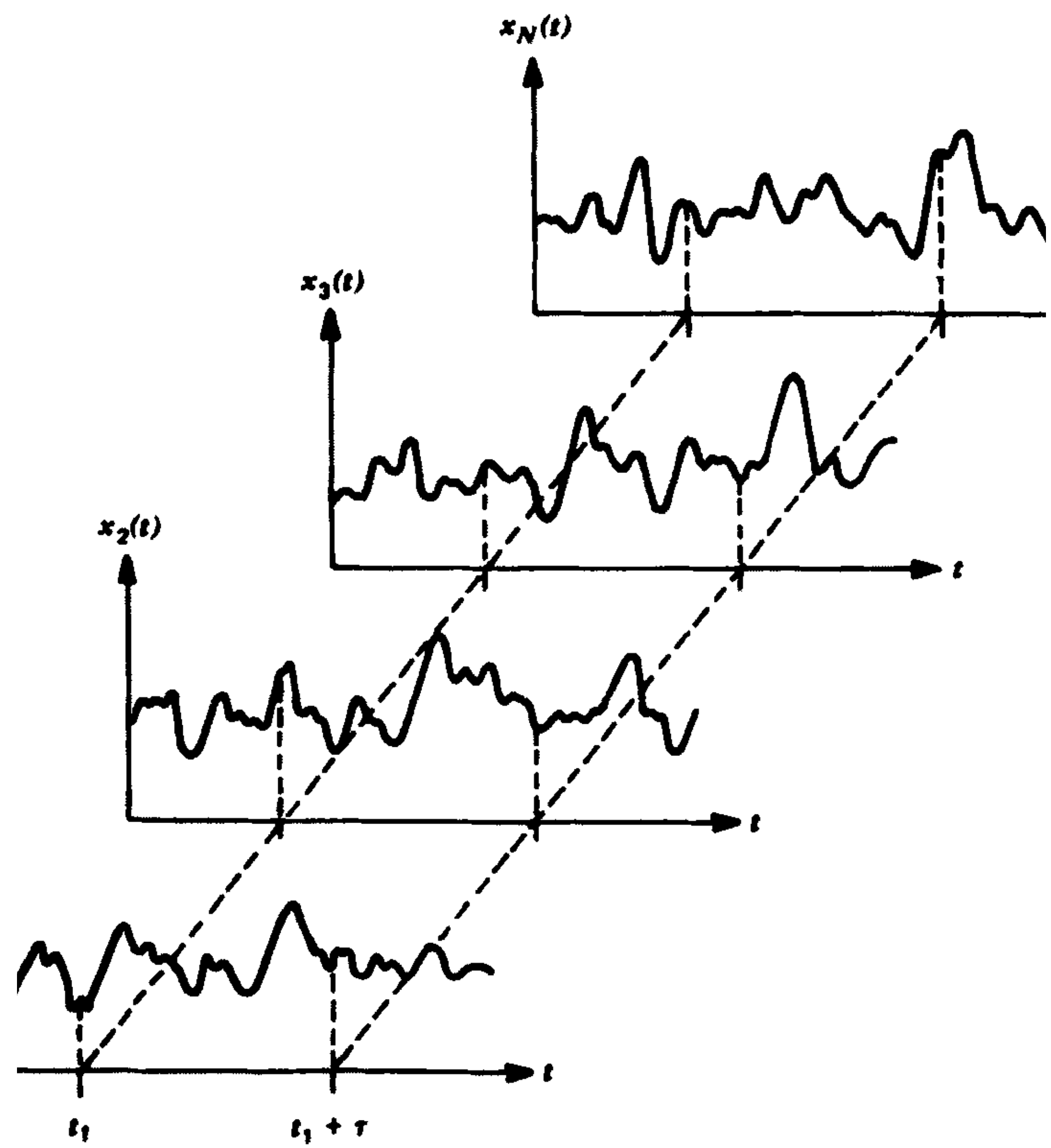
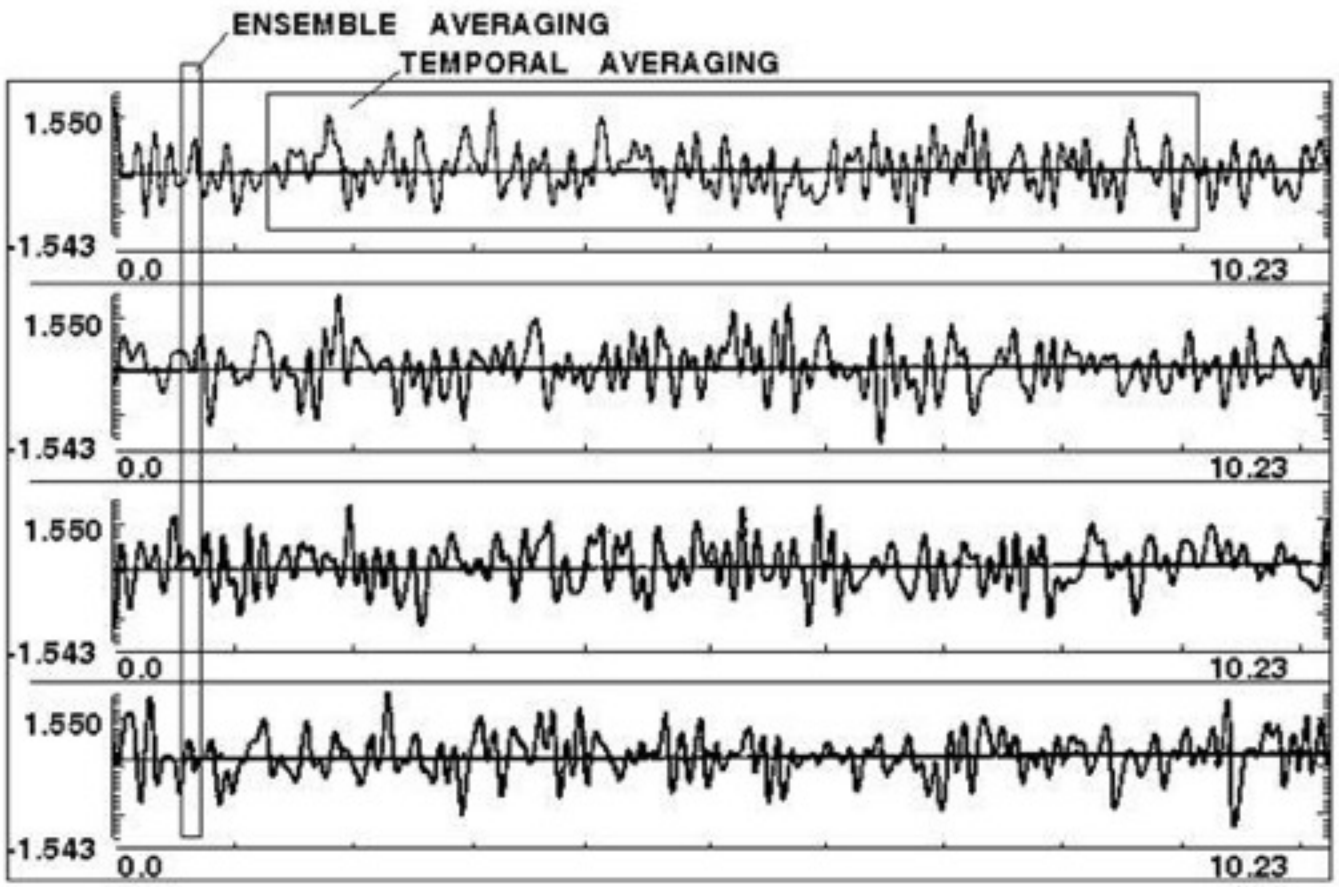


Figure 1.10 Ensemble of time history records defining a random process.

Ergodic Random Data

- A sample can be taken out of any signal, or across a signal and it will be representative of the event.
- This example could be turbulence across 4 flights in similar conditions with similar aircraft.



Analysis of Random Data

- Basic statistical properties of importance for describing single stationary random records are:
  - Mean, mean square values, and moments of order n
  - Probability density functions
  - Autocorrelation functions
  - Autospectral density functions
  - Joint probability density functions
  - Cross-correlation functions

Probability density functions

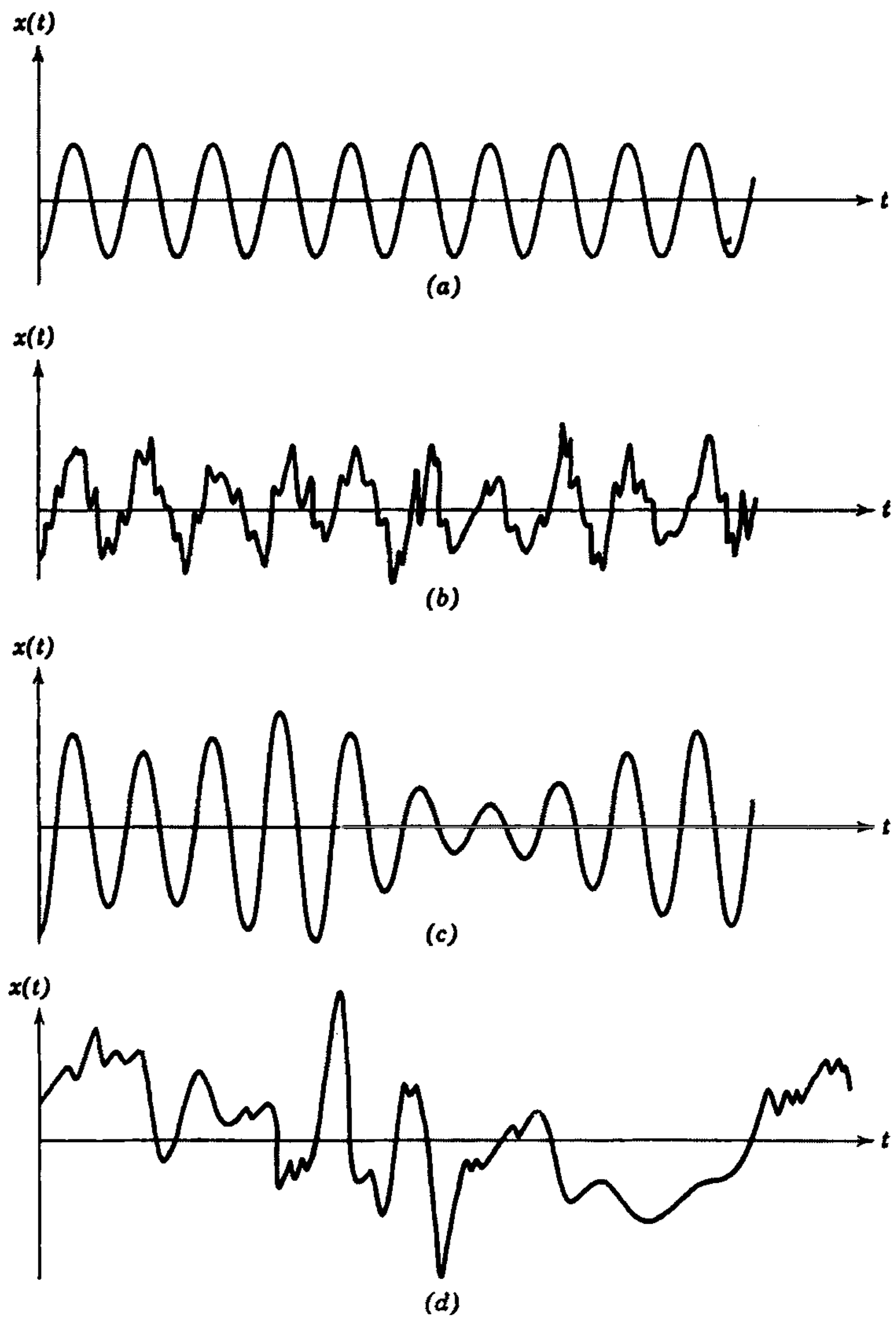


Figure 1.11 Four special time histories. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

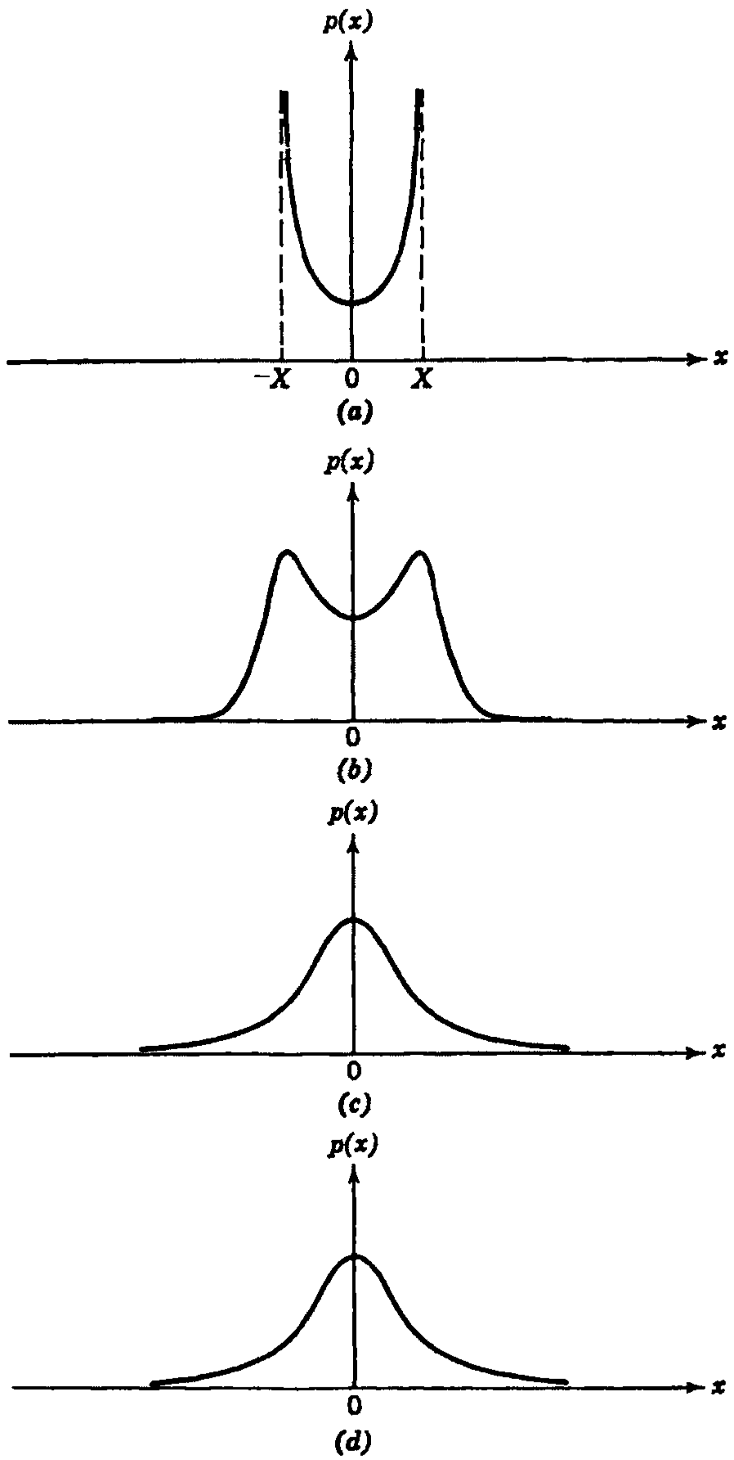


Figure 1.12 Probability density function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

Autocorrelation functions

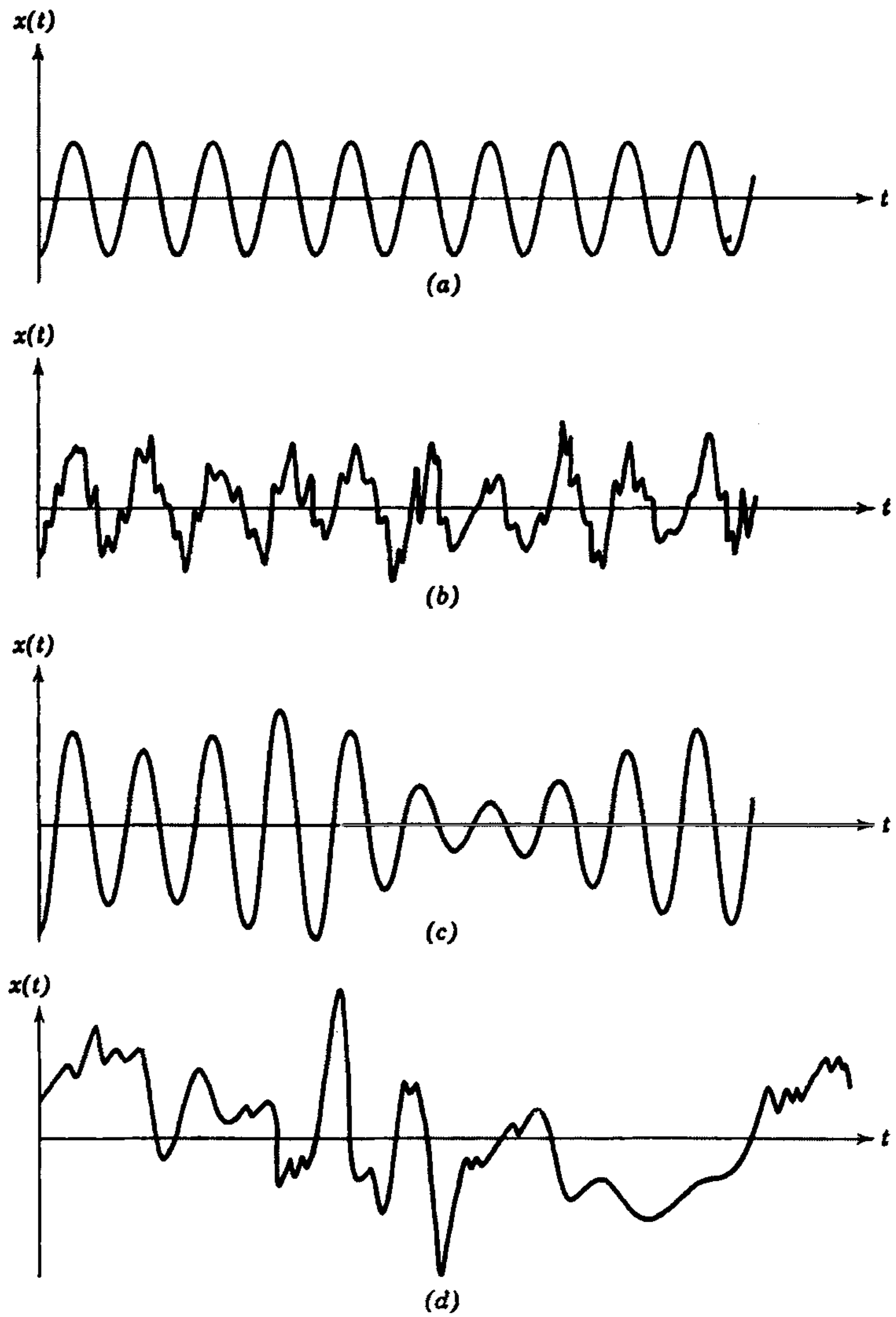


Figure 1.11 Four special time histories. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

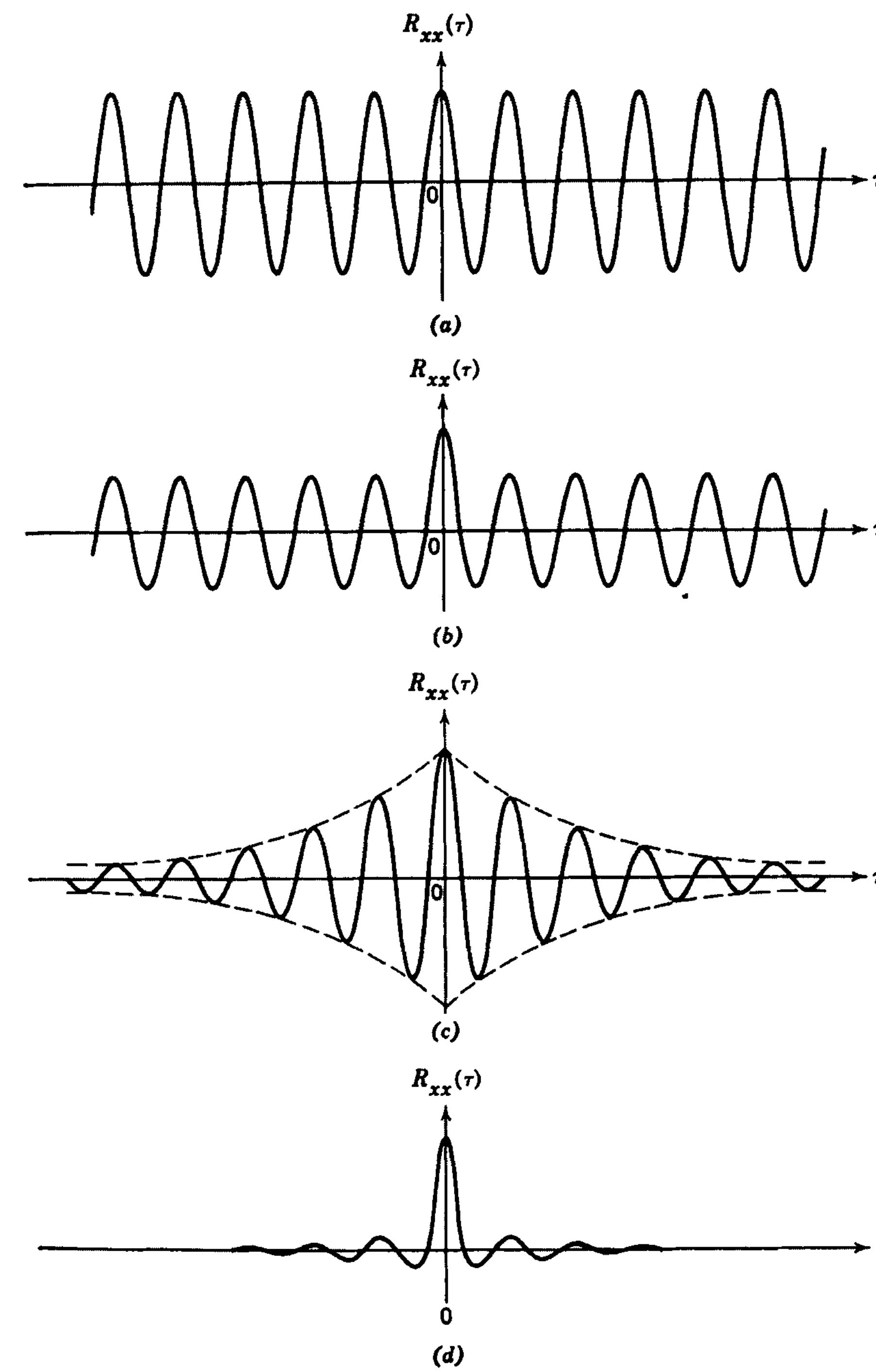


Figure 1.13 Autocorrelation function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

Autospectral density functions

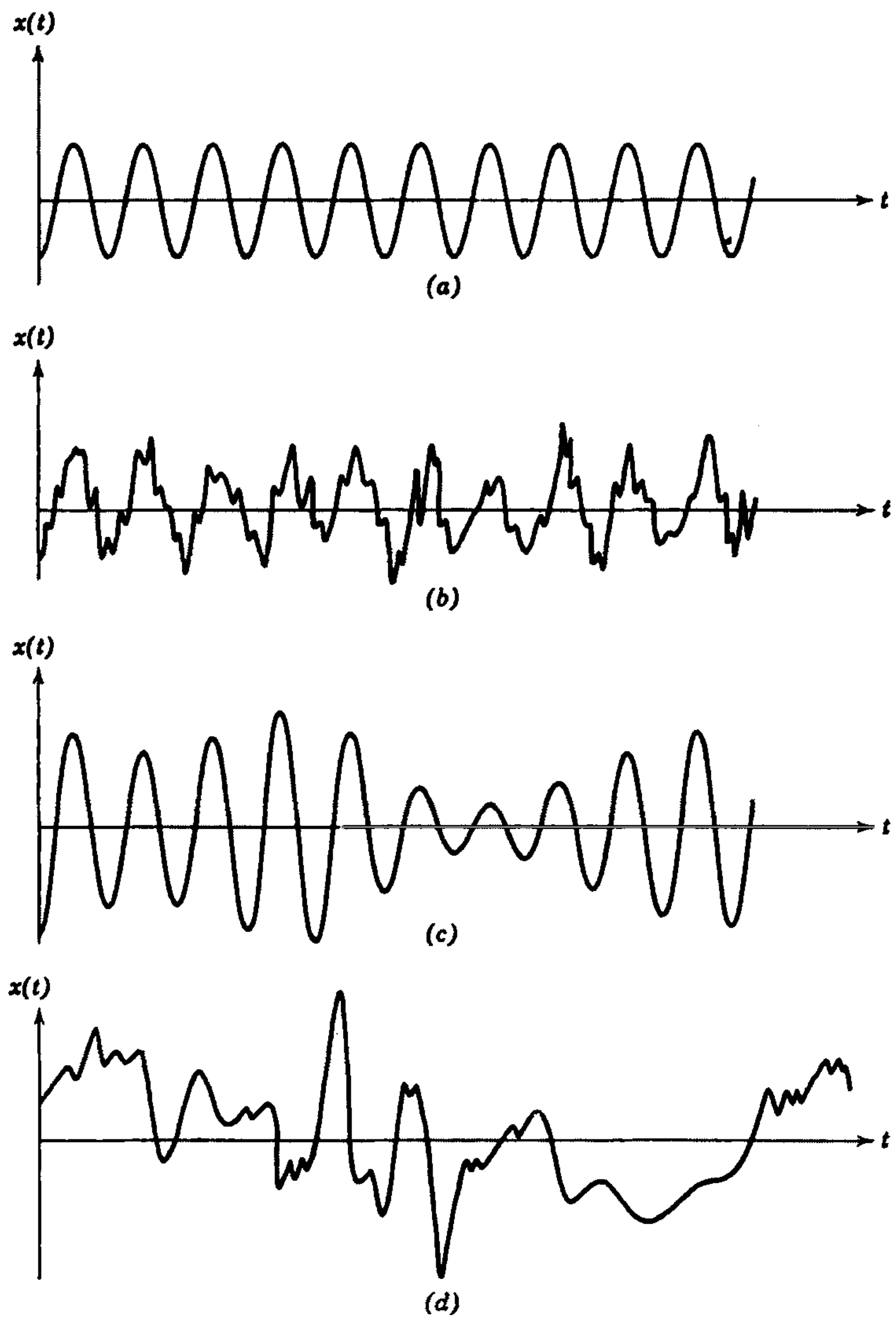


Figure 1.11 Four special time histories. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

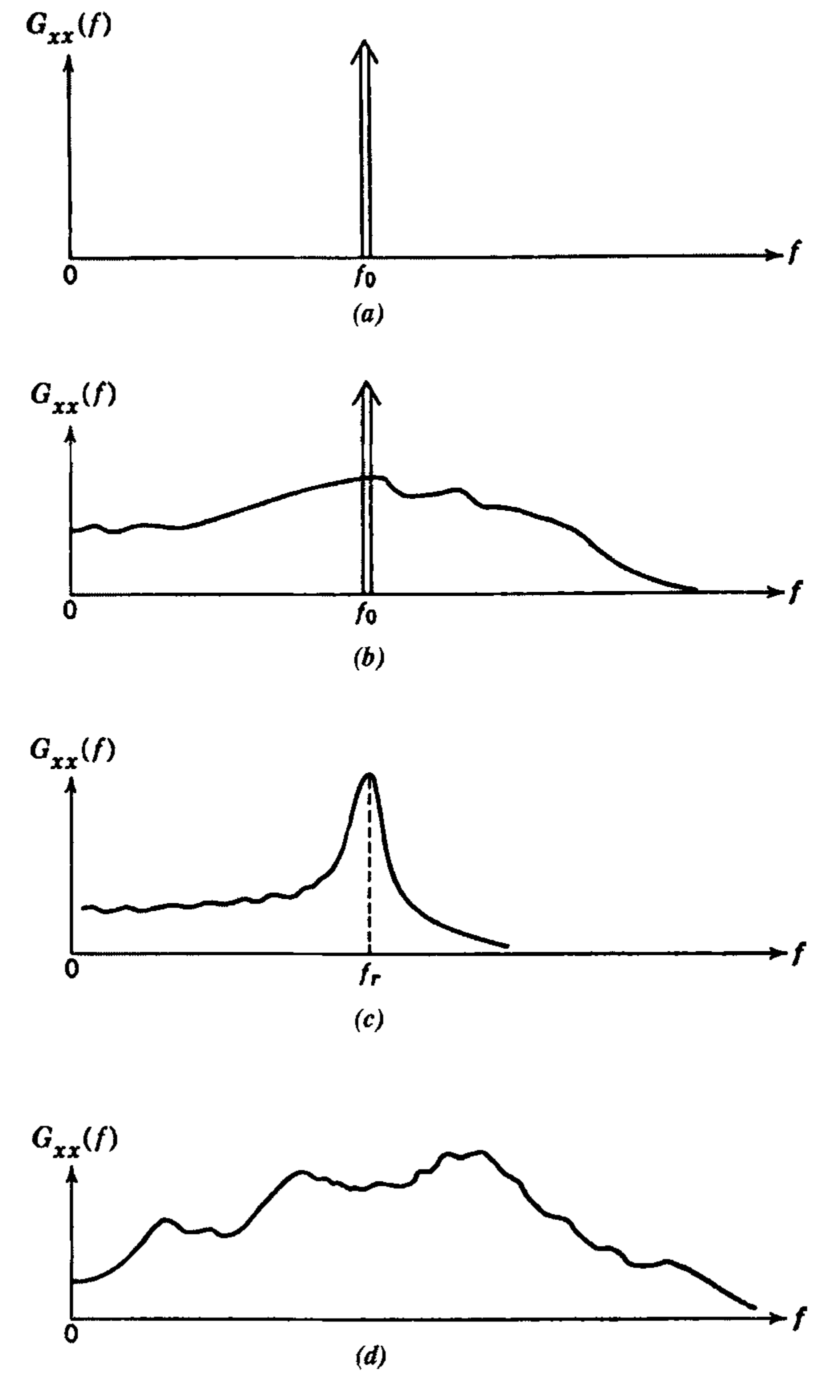
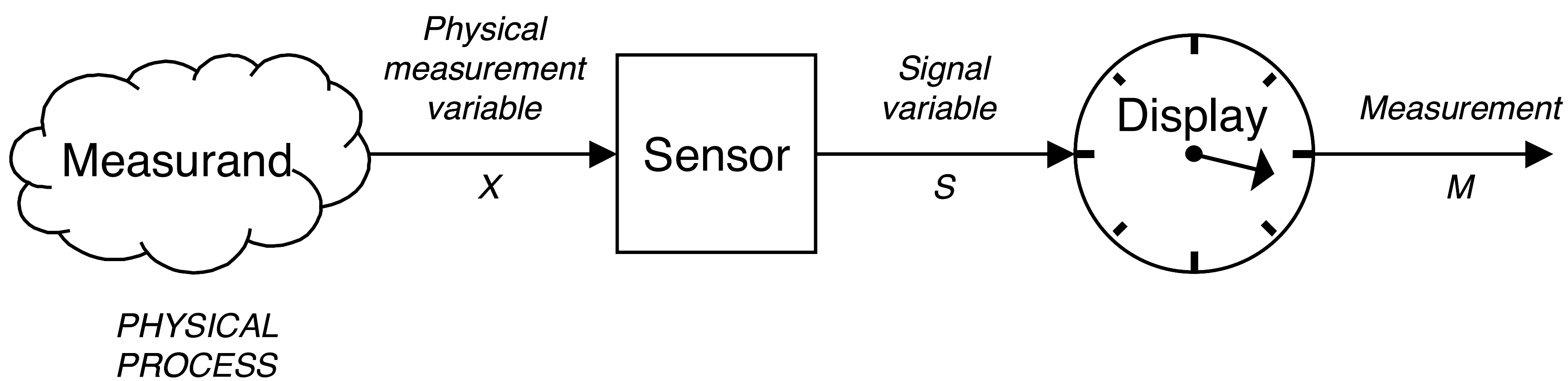


Figure 1.14 Autospectral density function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow bandwidth random noise. (*d*) Wide bandwidth random noise.

Characterization of Measurement Systems

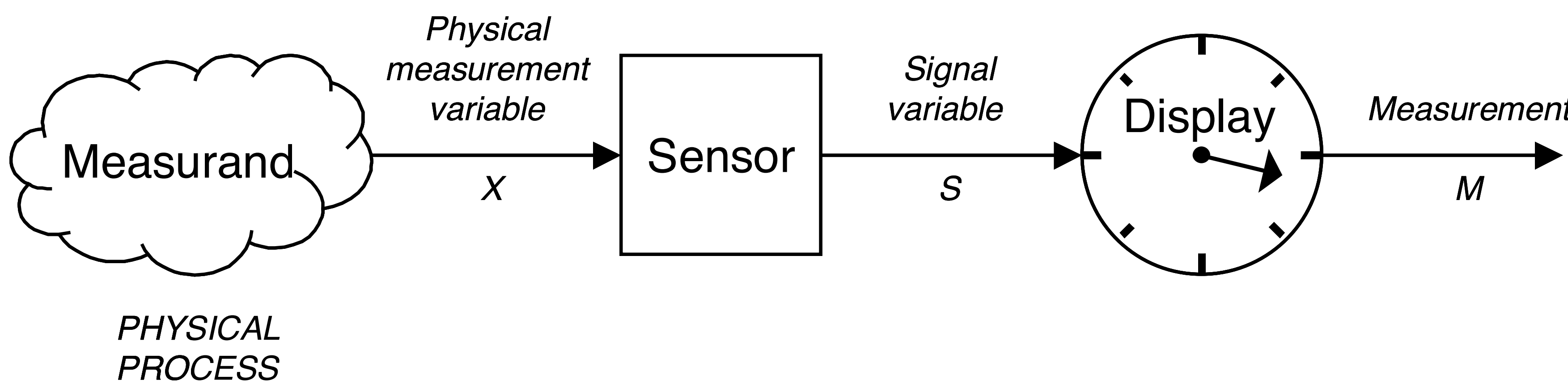
A simple instrument model



- An observable variable X is obtained from the measurand.
  - X is related to the measurand in some KNOWN way (i.e., measuring mass)
- The sensor generates a signal variable that can be manipulated:
  - Processed, transmitted or displayed
- In the example above the signal is passed to a display, where a measurement can be taken

Characterization of Measurement Systems

A simple instrument model



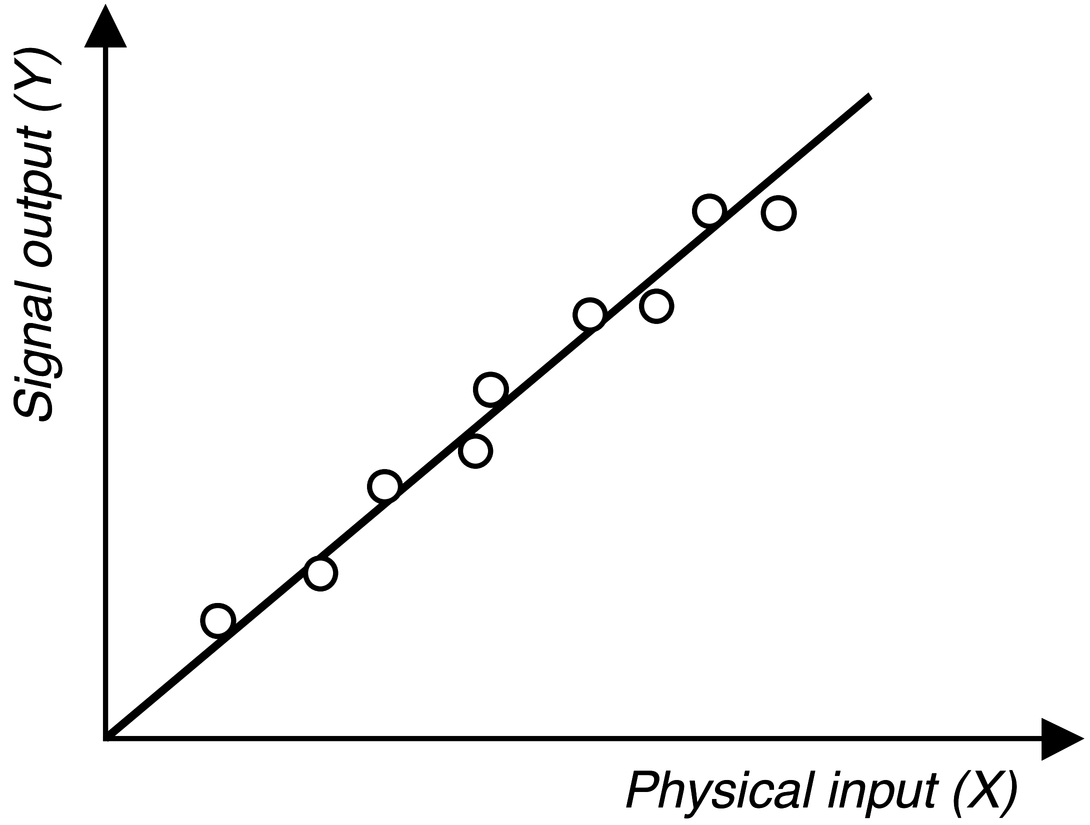
Measurement

- The process of comparing an unknown quantity with a standard of the same quantity (measuring length) or standards of two or more related quantities (measuring velocity)

Characterization of Measurement Systems

The relationship between the physical measurement variable (X) and the signal variable (S)

- A sensor or instrument is calibrated by applying a number of KNOWN physical inputs and recording the response of the system.

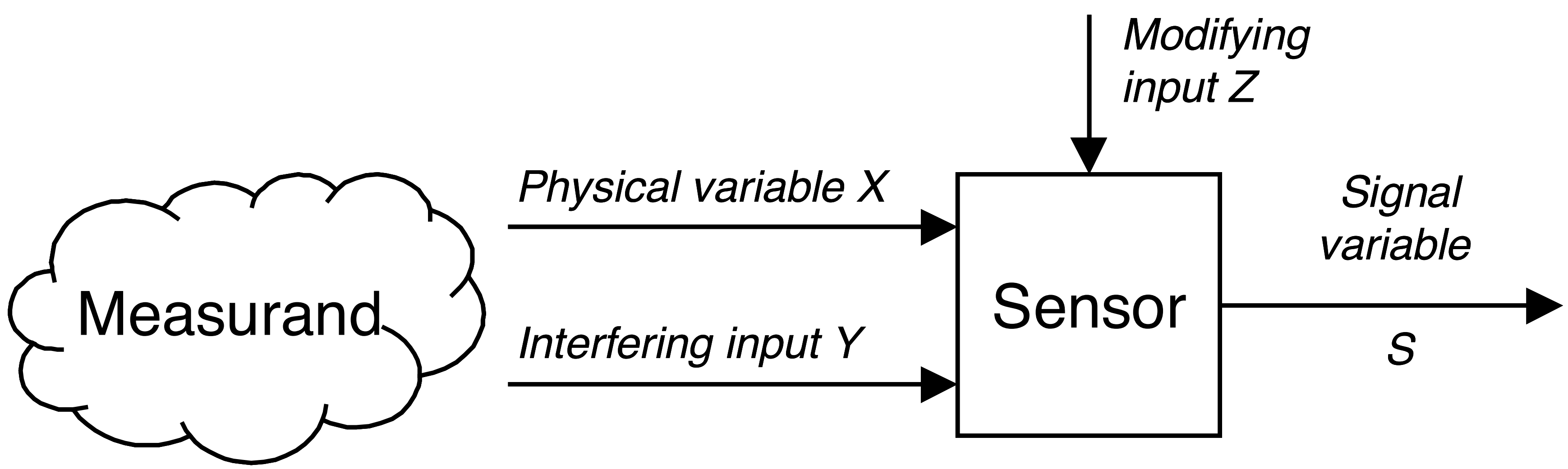


Characterization of Measurement Systems

Interfering inputs (Y)

- Those that the sensor to respond as the linear superposition with the measurand variable X.

  - Linear superposition assumption: S(aX +bY)=aS(X)+bS(Y)

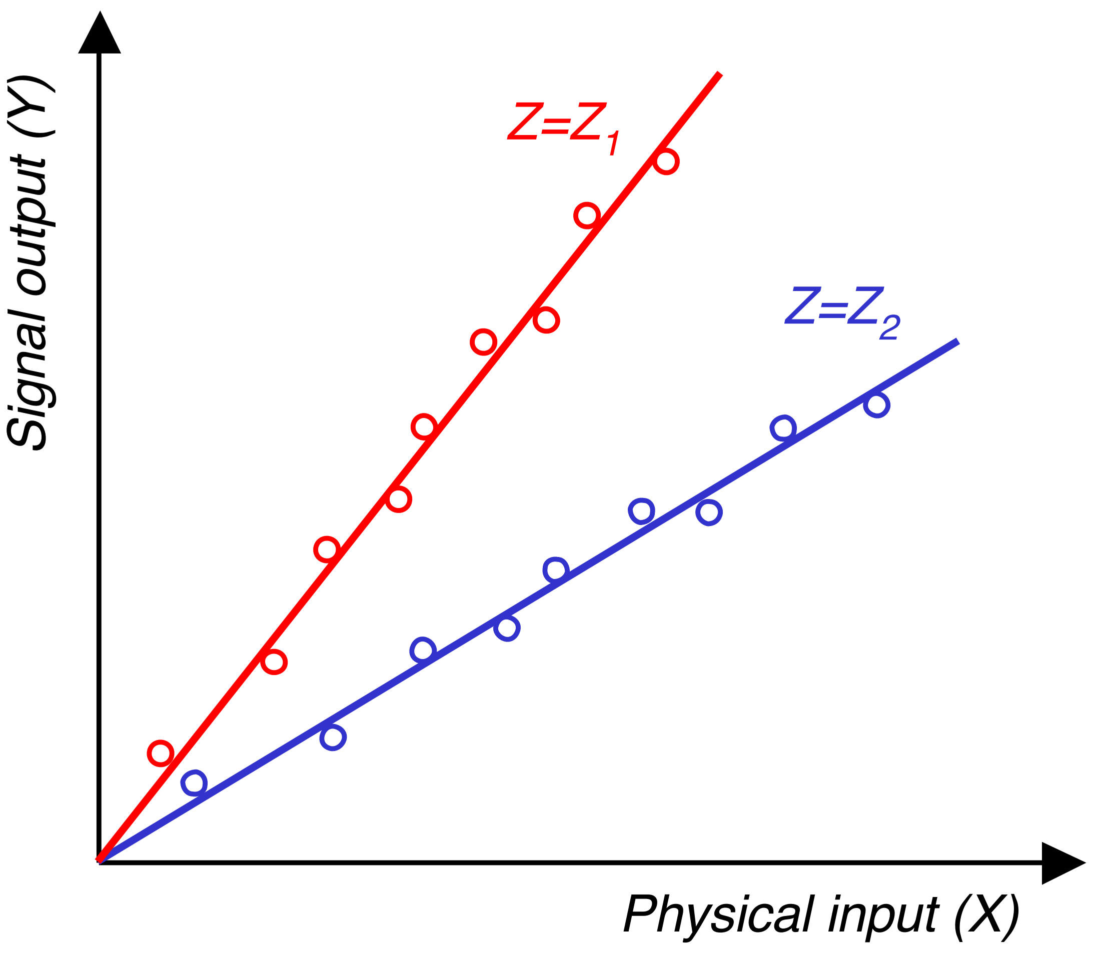


Characterization of Measurement Systems

Modifying inputs (Z)

- Those that change the behavior of the sensor and, hence, the calibration curve

  - Temperature is a typical modifying input.



Characterization of Measurement Systems

Static characteristics

- The properties of the system after all transient effects have settled to their final or steady state.
  - Accuracy
  - Discrimination
  - Precision
  - Errors
  - Drift
  - Sensitivity
  - Linearity
  - Hystheresis

Characterization of Measurement Systems

Dynamic characteristics

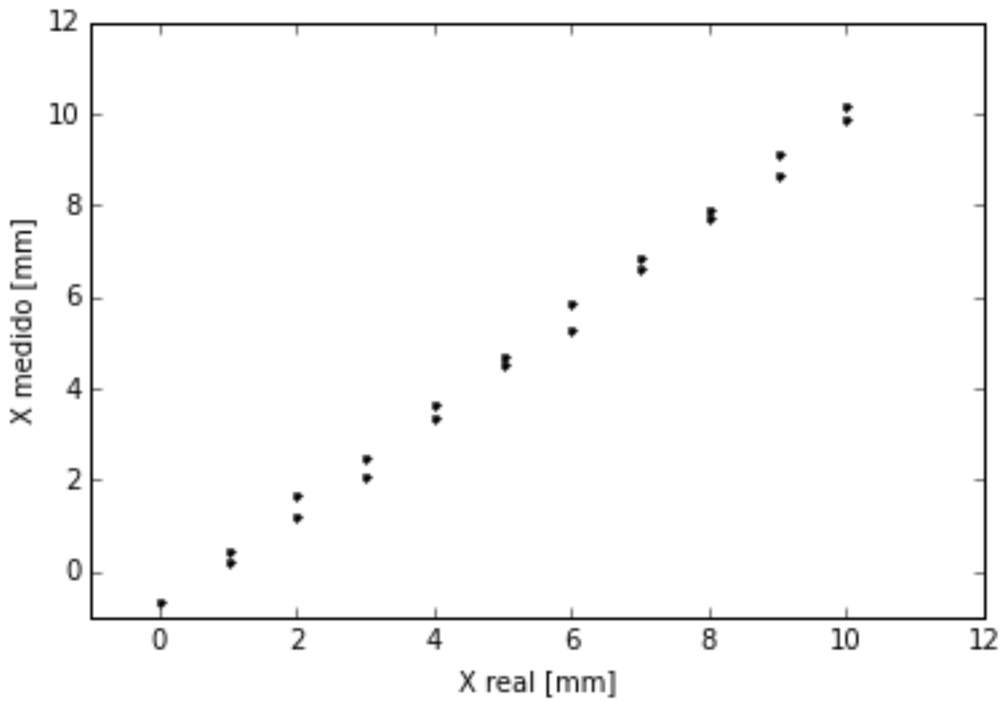
- The properties of the system transient response to an input.
  - Zero order systems.
  - First order systems.
  - Second order systems.

Ejemplo - Calibración

- **Ejemplo.** Un sistema de medida de altura usando pulsos de luz. La tabla muestra los valores reales y los medidos (con error) cuando se incrementa la distancia y cuando se disminuye.

X real(mm)	X medido (Inc.)	X medido (Dism.)
0	-1.12	-0.69
1	0.21	0.42
2	1.18	1.65
3	2.09	2.48
4	3.33	3.62
5	4.50	4.71
6	5.26	5.87
7	6.59	6.86
8	7.73	7.92
9	8.68	9.10
10	9.88	10.20

Ejemplo - Calibración



X real(mm)	X medido (Inc.)	X medido (Dism.)
0	-1.12	-0.69
1	0.21	0.42
2	1.18	1.65
3	2.09	2.48
4	3.33	3.62
5	4.50	4.71
6	5.26	5.87
7	6.59	6.86
8	7.73	7.92
9	8.68	9.10
10	9.88	10.20

Combination of errors

- In general when *f* is a function of *x,y,z,*

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \cdots$$

Table 3.1 *Propagation of standard uncertainties in combined quantities or functions.*

$f = x + y$ or $f = x - y$	$\sigma_f^2 = \sigma_x^2 + \sigma_y^2$
$f = xy$ or $f = x/y$	$(\sigma_f/f)^2 = (\sigma_x/x)^2 + (\sigma_y/y)^2$
$f = xy^n$ or $f = x/y^n$	$(\sigma_f/f)^2 = (\sigma_x/x)^2 + n^2(\sigma_y/y)^2$
$f = \ln x$	$\sigma_f = \sigma_x/x$
$f = e^x$	$\sigma_f = f\sigma_x$