Interferometry

Optical Metrology

LIGHT WAVES

- Light can be thought of as a transverse electromagnetic wave propagating through space.
- The electric and magnetic fields are linked to each other and propagate together.
- It is usually sufficient to consider only the electric field at any point.

$$\vec{E}(z,t) = \vec{E}_0 cos\left[2\pi\left(\nu t - \frac{z}{\lambda}\right)\right]$$

where *E* is the amplitude of the light wave, *ν* is its frequency, and *λ* is its wavelength.

$$\vec{E}(x,y,z,t) = \vec{E}_0 cos\left[wt - \vec{k}.\vec{r}\right]$$

$$\vec{E}(x,y,z,t) = Re\left\{\vec{E}_0 exp\left(-j\vec{k}.\vec{r}\right) exp(jwt)\right\}$$

Complex exponential representation

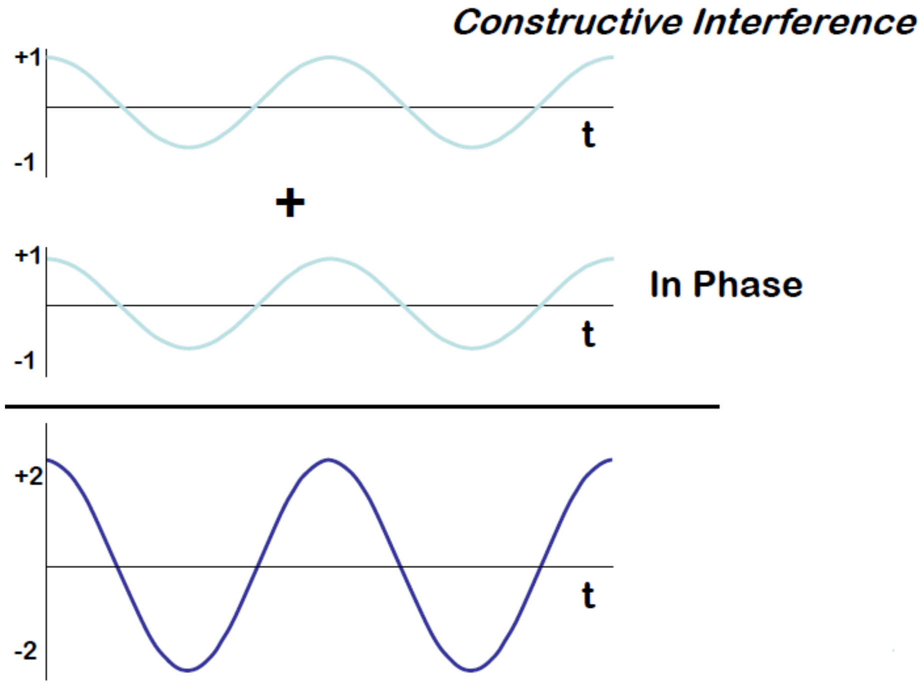
$$\vec{E}(x,y,z,t) = Re\left\{\vec{A} exp(jwt)\right\}$$

THE INTENSITY

The most common detectors (Eye, photodiodes, multiplication tubes, photographic film, etc.) register the irradiance, which is proportional to the field amplitude absolutely squared

$$I = \frac{\varepsilon v}{2} U^2$$

Superposition



$$\vec{E}_T = \vec{E}_1 + \vec{E}_2$$

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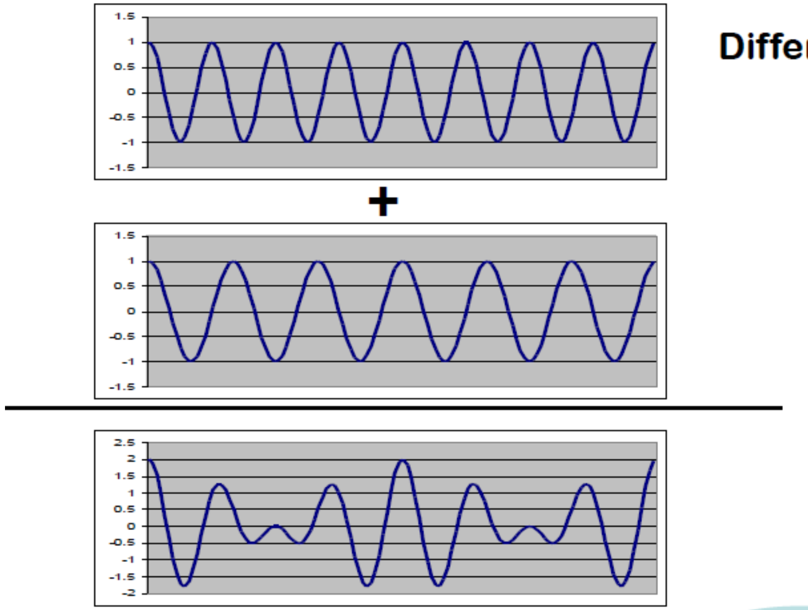

Different f

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2$$

1) Constructive **2) Destructive** **3) Neither**

Interference

- Interference can occur when two or more waves overlap each other in space.

- The electromagnetic wave theory tells us that the resulting field simply becomes the superposition.

- Resulting intensity is not just ($I_1 + I_2$).

Interference

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Assume that two waves described by

$$u_1 = U_1 \mathrm{e}^{\mathrm{i}\phi_1}$$

$$u_2 = U_2 \mathrm{e}^{\mathrm{i}\phi_2}$$

Superposition

$$u = u_1 + u_2$$

$$I = |u|^2 = |u_1 + u_2|^2 = U_1^2 + U_2^2 + 2U_1U_2\cos(\phi_1 - \phi_2)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$\Delta\phi = \phi_1 - \phi_2$$

Interference

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$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$\Delta\phi = \phi_1 - \phi_2 \quad \begin{cases} \Delta\phi = (2n+1)\pi, \quad \text{for} \quad n = 0, 1, 2, \ldots & \text{Destructive interference} \\ \Delta\phi = 2n\pi, \quad \text{for} \quad n = 0, 1, 2, \ldots & \text{Constructive interference} \end{cases}$$

Coherence

- Detection of light is an averaging process in space and time.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma(\tau)| \cos \Delta\phi$$

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

contrast or visibility

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma(\tau)|$$
$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}|\gamma(\tau)|$$

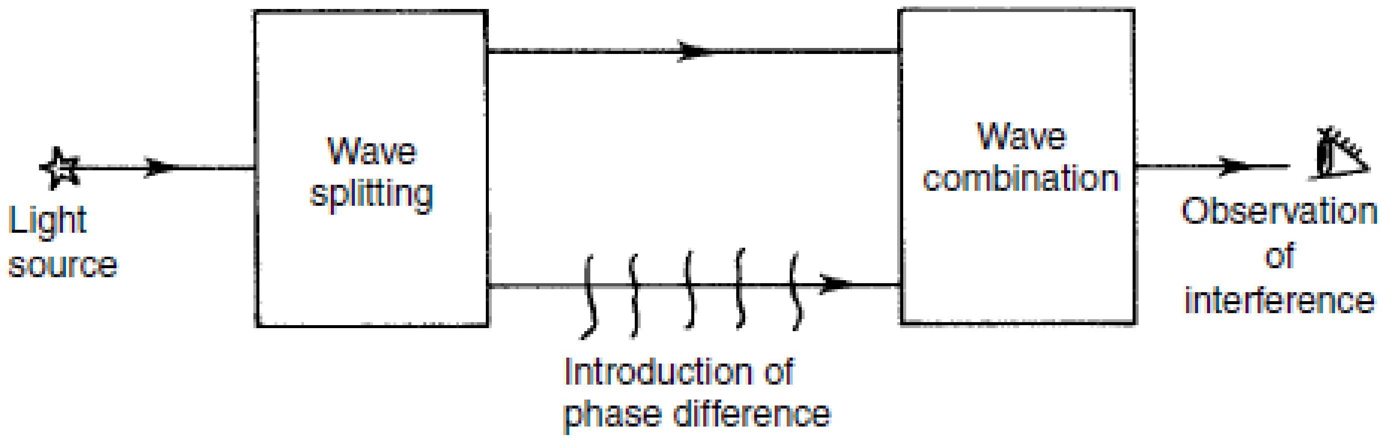
$$V = \frac{2\sqrt{I_1 I_2}|\gamma(\tau)|}{I_1 + I_2}$$

$$|\gamma(0)| = 1$$
$$|\gamma(\tau_c)| = 0$$
$$0 \leq |\gamma(\tau)| \leq 1$$

Interferometry

- By measuring the distance between interference fringes over selected planes in space, quantities such as the angle and distance can be found.
- One further step would be to apply for a wave reflected from a rough surface.
- By observing the interference -can determine the surface topography.
- For smoother surfaces, however, such as optical components (lenses, mirrors, etc.) where tolerances of the order of fractions of a wavelength are to be measured, that kind of interferometry is quite common.

Interferometer



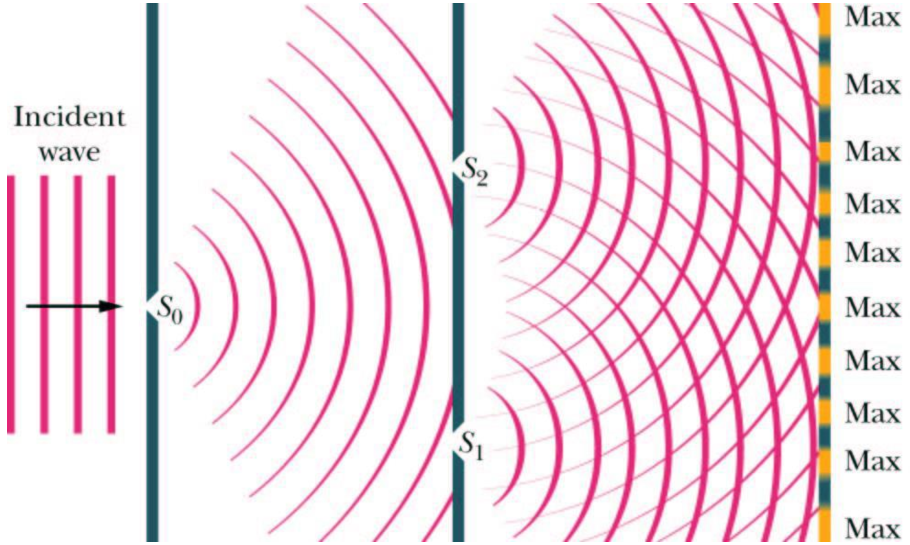
Most interferometers have the following elements:

- Light source.
- Element for splitting the light into two (or more) partial waves.
- Different propagation paths where the partial waves undergo different phase contributions.
- Element for superposing the partial waves.
- Detector for observation of the interference.

Interferometer

Depending on how the light is split, interferometers are commonly classified .

- Wavefront division interferometers.
- Amplitude division interferometers.

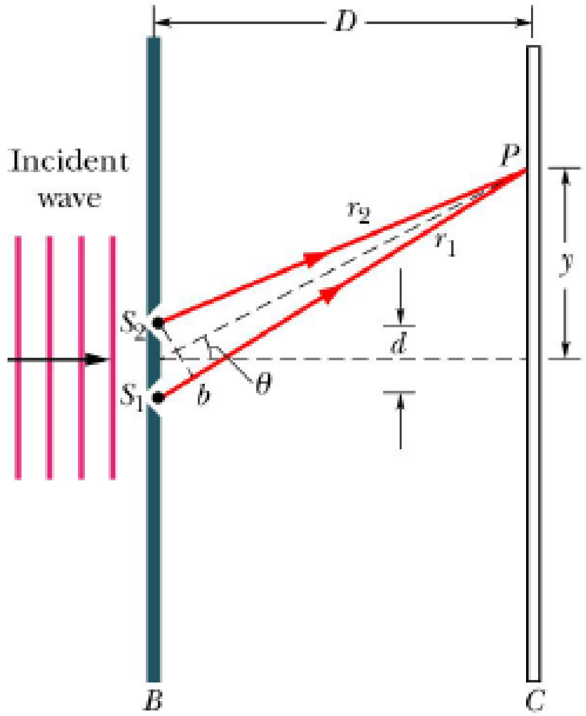


Wavefront Division Interferometers

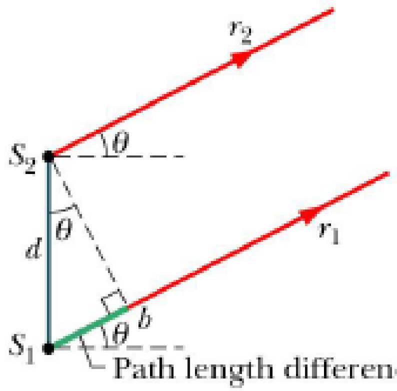
Example of a wavefront dividing interferometer, (Thomas Young)

- The incident wavefront is divided by passing through two small holes at S_l and S_2 in a screen 1.
- The emerging spherical wavefronts from S_l and S_2 will interfere, and the pattern is observed on screen 2.
- The path length differences of the light reaching an arbitrary point x on S_2 is found from Figure.
- When the distance D between screens is much greater than the distance d between S_1 and S_2, we have a good approximation

Wavefront Division



(a)



$= d\sin\theta$

(b)

$$d\sin\theta = m\lambda \quad \mathrm{m}=0,1,2,3\ldots \ \text{Maximum}$$

$$\tan\theta = \frac{y_m}{D} \quad \text{or} \quad y_m = D\tan\theta \approx D\sin\theta$$

$$y_m = \frac{m\lambda D}{d}$$

Maxima

m	y_m +/-
0	0
1	Dλ/d
2	2Dλ/d
3	3Dλ/d

Minima

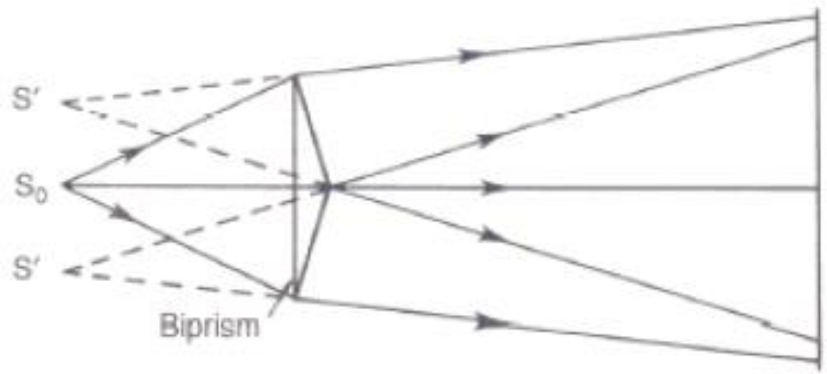
m	y_m +/-
0	Dλ/2d
1	3Dλ/2d
2	5Dλ/2d
3	7Dλ/2d

$$d\sin\theta = (m + \frac{1}{2})\lambda \quad \mathrm{m}=0,1,2,3\ldots \ \text{Minimum}$$

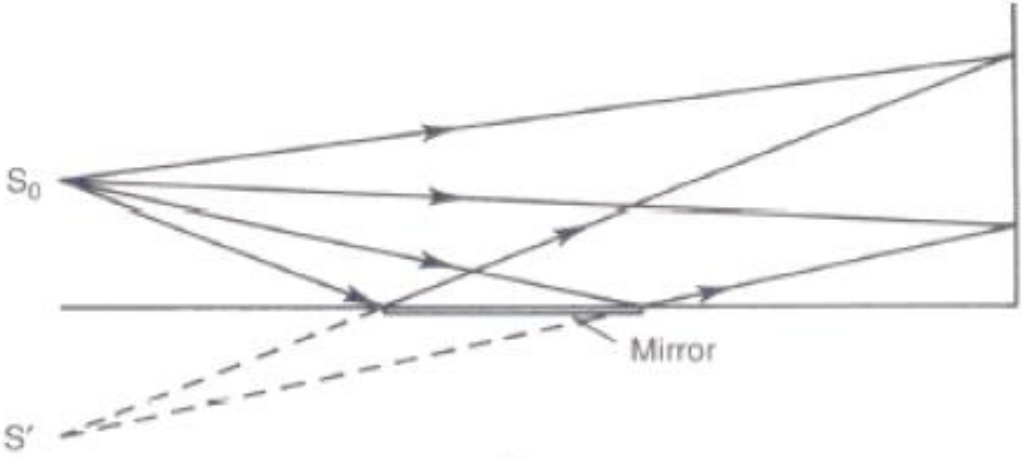
$$y_m = \frac{(m+1/2)\lambda D}{d}$$

Wavefront Division

Other interferometers



(a)

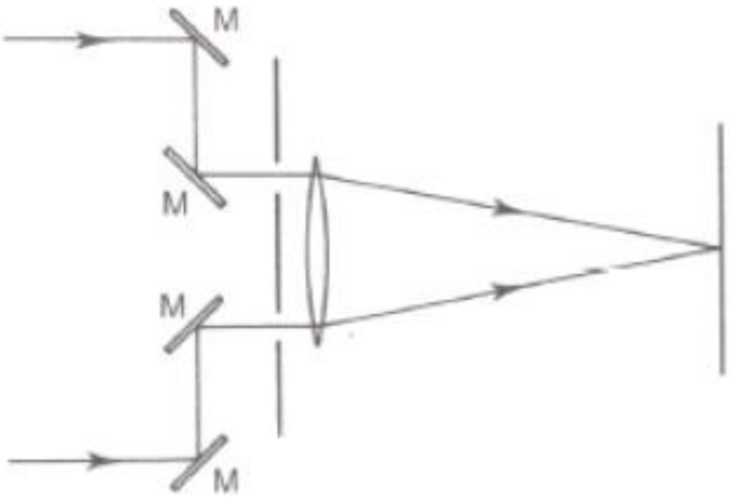


(b)

A) Fresnel Biprism

B) Lloyds Mirror

C) Michelsons Stellar Interferometer

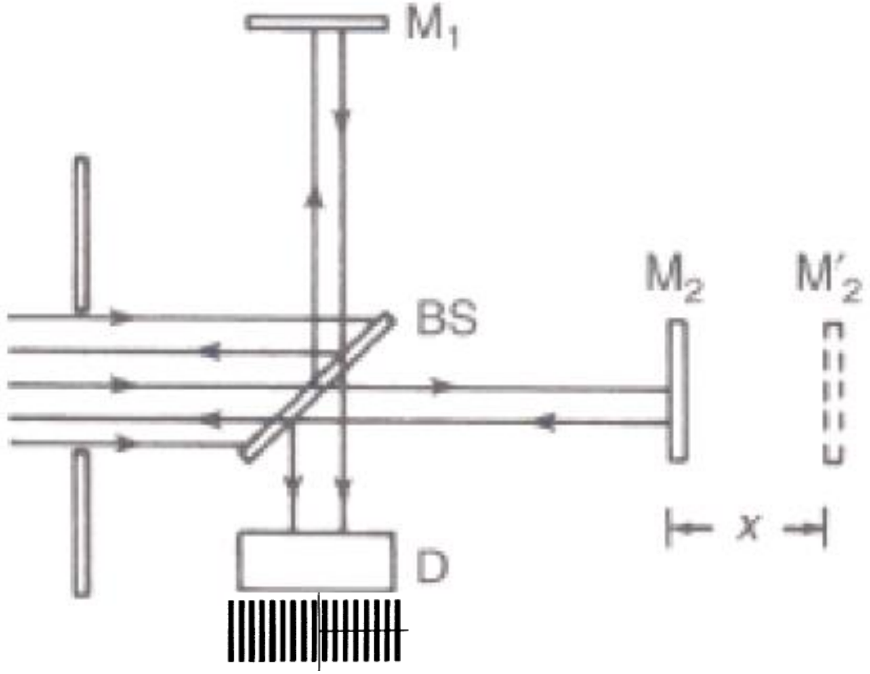


(c)

Amplitude Division Interferometers

Example of a amplitude dividing interferometer, (Michelson)

- Amplitude is divided by beam splitter BS which partly reflects and partly transmits.
- These divided light go to two mirrors M_1 and M_2 where they are reflected back.
- The reflected lights recombine to form interference on the detector D.
- The path length can be varied by moving one of the mirrors or by mounting that on movable object (movement of x give path difference of 2x) and phase difference: $\Delta\phi = \frac{2\pi}{\lambda} 2x$

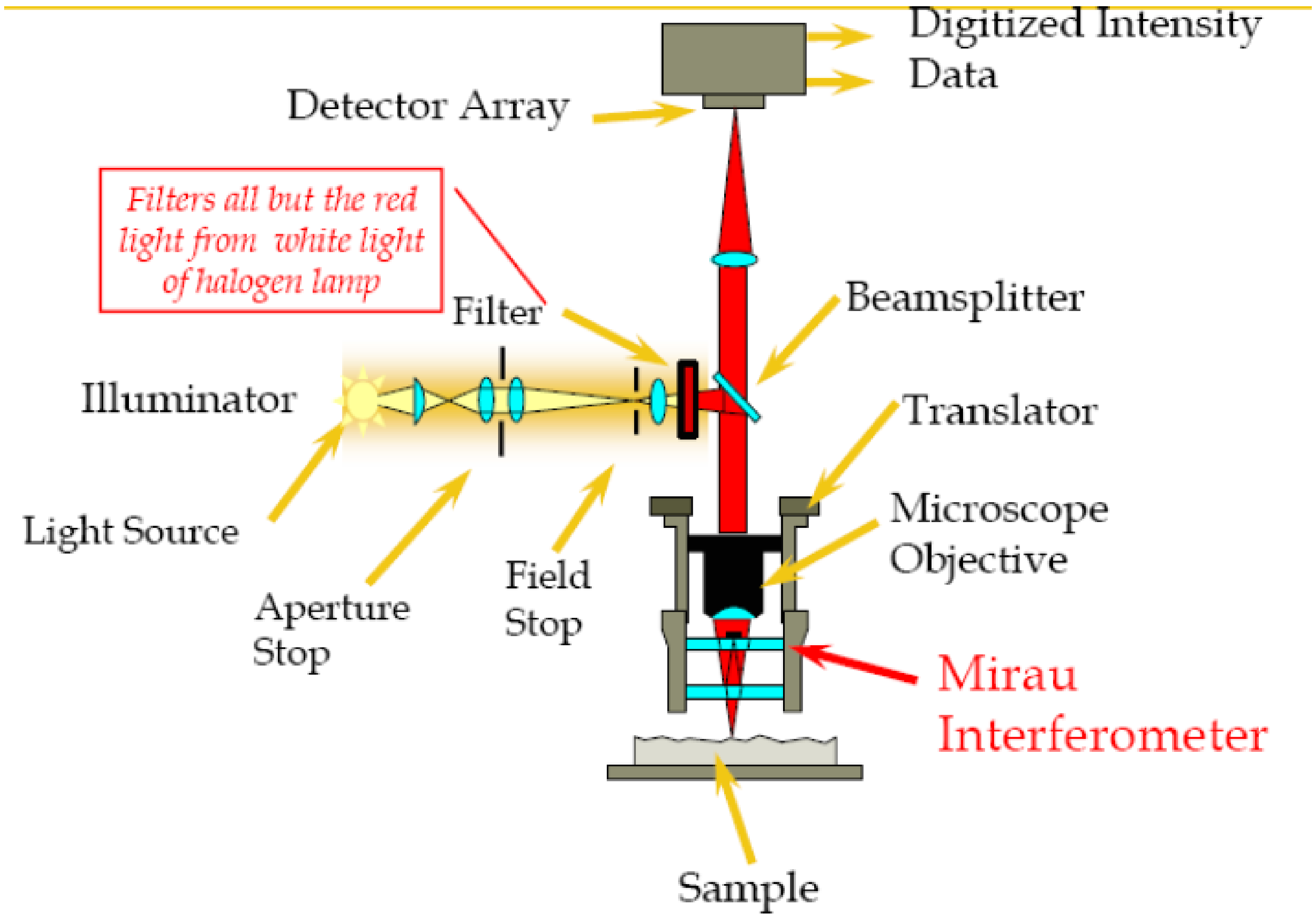


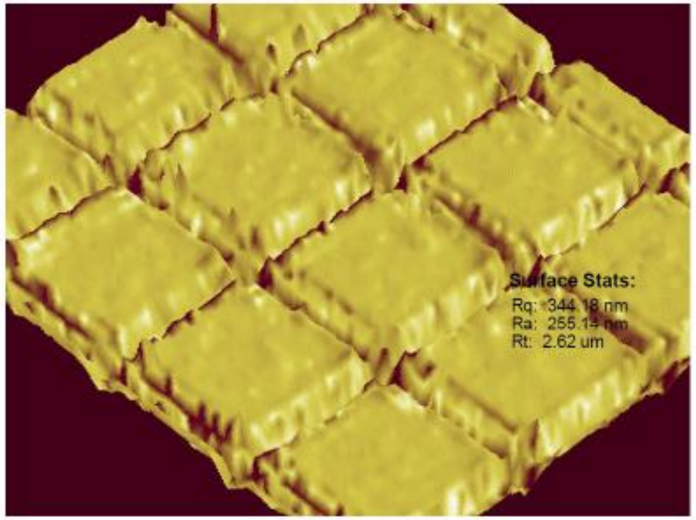
- As M_2 moves the displacement is measured by counting the number of light maxima registered by D.

- By counting the number of maxima per unit time will give the velocity of the object.

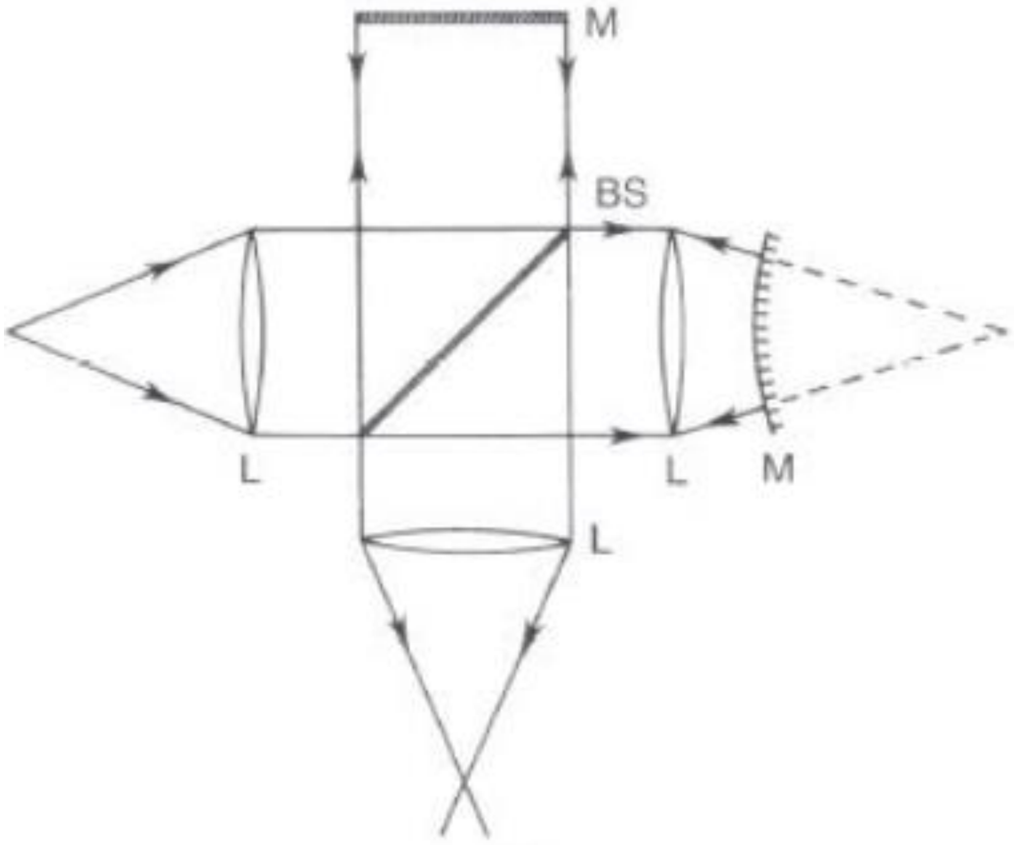
$$I(x) = 2I\left(1 + \cos\frac{4\pi x}{\lambda}\right)$$

Mirau Interferometer

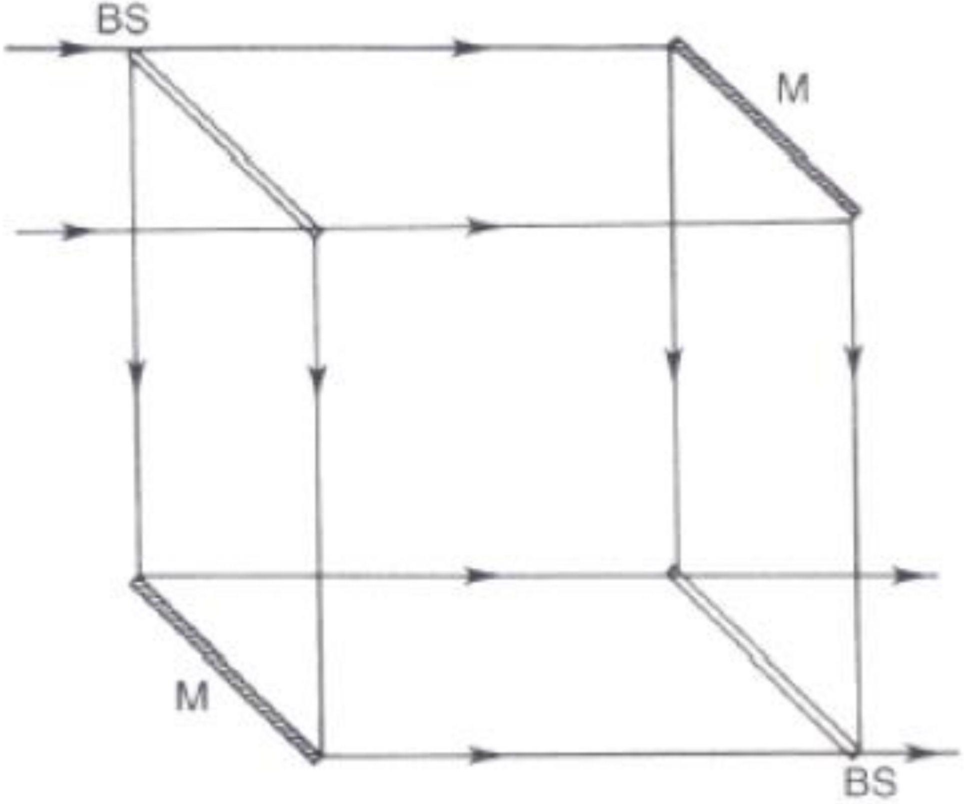




Amplitude Division Interferometers



(a)



(b)

- Twyman Green Interferometer
- Mach Zehnder Interferometer

Temporal Coherence

$$I = \Delta I\left(1+\cos\left(\frac{2\pi\nu d}{c}\right)\right) = \Delta I\left(1+\cos(2\pi\nu\tau)\right)$$

$$I = \sum_n I_n\left(1+\cos(2\pi\nu_n\tau)\right)$$

$$P(\nu) = S(\nu-\nu_0)$$

Normalized spectral distribution function of the source

$$I = \int_0^\infty I(\nu)\left(1+\cos(2\pi\nu\tau)\right)\mathrm{d}\nu = I_0\left[1+\int_0^\infty P(\nu)\cos(2\pi\nu\tau)\,\mathrm{d}\nu\right]$$

$$I_0 = \int_0^\infty I(\nu)\,\mathrm{d}\nu$$

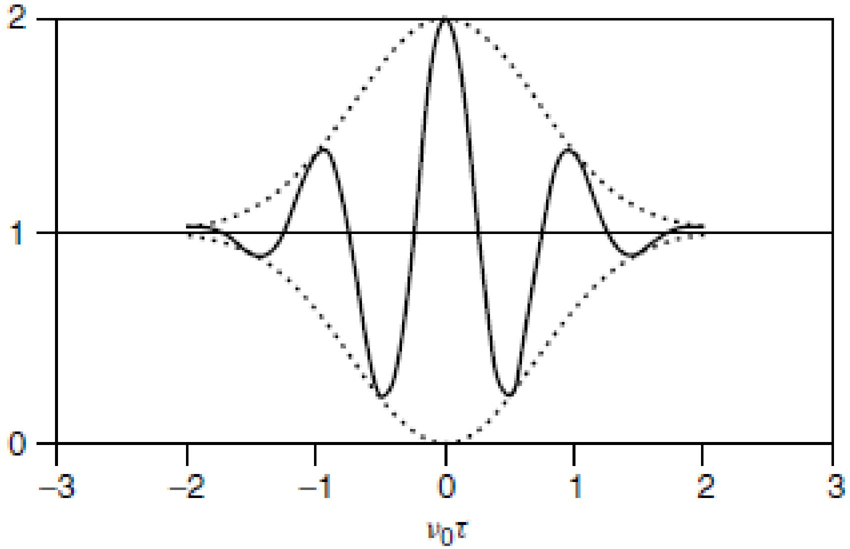
$$\begin{aligned}\int_0^\infty P(\nu)\cos(2\pi\nu\tau)\,\mathrm{d}\nu &= \mathrm{Re}\left\{\int_{-\infty}^{\infty} S(\nu-\nu_0)\mathrm{e}^{-\mathrm{i}2\pi\nu\tau}\,\mathrm{d}\nu\right\}\\ &= \mathrm{Re}\left\{\mathrm{e}^{-\mathrm{i}2\pi\nu_0\tau}\int_{-\infty}^{\infty} S(\nu)\mathrm{e}^{-\mathrm{i}2\pi\nu\tau}\,\mathrm{d}\nu\right\}\\ &= |\gamma(\tau)|\cos(2\pi\nu_0\tau-\varphi)\end{aligned}$$

Temporal Coherence

$$\int_0^\infty P(\nu)\cos(2\pi\nu\tau)\,\mathrm{d}\nu = \mathrm{Re}\left\{\int_{-\infty}^{\infty} S(\nu-\nu_0)\mathrm{e}^{-\mathrm{i}2\pi\nu\tau}\,\mathrm{d}\nu\right\}$$
$$= \mathrm{Re}\left\{\mathrm{e}^{-\mathrm{i}2\pi\nu_0\tau}\int_{-\infty}^{\infty} S(\nu)\mathrm{e}^{-\mathrm{i}2\pi\nu\tau}\,\mathrm{d}\nu\right\}$$
$$= |\gamma(\tau)|\cos(2\pi\nu_0\tau-\varphi)$$

$$I = I_0[1+|\gamma(\tau)|\cos(2\pi\nu_0\tau-\varphi)]$$

$$\gamma(\tau) = \int_{-\infty}^{\infty} S(\nu)\mathrm{e}^{-\mathrm{i}2\pi\nu\tau}\,\mathrm{d}\nu = |\gamma(\tau)|\mathrm{e}^{\mathrm{i}\varphi}$$



Source with a Gaussian spectral distribution function

$$P(\nu) = \frac{2\sqrt{\ln 2}}{\sqrt{\pi}\,\Delta\nu}\exp\left\{-\left(2\sqrt{\ln 2}\frac{\nu-\nu_0}{\Delta\nu}\right)^2\right\}$$

$$I = I_0\left[1+\exp\left\{-\left(\frac{\pi\,\Delta\nu\tau}{2\sqrt{\ln 2}}\right)^2\right\}\cos(2\pi\nu_0\tau)\right]$$

$$\tau_c = \sqrt{\frac{2\ln 2}{\pi}}\frac{1}{\Delta\nu} = \frac{0.664}{\Delta\nu}$$

The coherence length $L_c = c\tau_c$

Phase-Shifting Interferometry (PSI)

One of the most used methods for phase extraction is based on a phase change between the interference beams by a known value, while their amplitudes are keeping constant.

phase-shifting interferometry, phase-sampling interferometry, or phase-stepping interferometry.

$$I = a + b\cos(\phi + \psi)$$

A reference wave front is moved along its propagation direction respecting to the probe wave front changing with this the phase difference between them.

Phase-Shifting Interferometry (PSI)

Phase extraction methods

A shift of ψ_0 is made for *N* steps, then *N* intensity values I_n will be measured, (where *n* =1,...,*N*)

$$I_n = a + b\cos(\phi + \psi_{0n})$$

Where $\psi_{0n} = 2\pi n / N$

$$I_n = A + B\cos\psi_{0n} + C\sin\psi_{on}$$

$$A = a\ ;\quad B = b\cos\phi\ ;\quad C = -b\sin\phi$$

$$B = \frac{2}{N}\sum_{n=1}^{N} I_n \cos\psi_{0n}\ ;\quad C = \frac{2}{N}\sum_{n=1}^{N} I_n \sin\psi_{0n}$$

$$\phi = \tan^{-1}\frac{-C}{B} = \tan^{-1}\frac{\sum I_n \sin\psi_{0n}}{\sum I_n \cos\psi_{0n}}$$

Three steps technique

we need a minimum of three interferograms to reconstruct the wavefront, the phase can be calculated with a phase-shift of $\frac{\pi}{2}$ per exposition.

$$I_1 = a + b\cos\left(\phi + \frac{1}{4}\pi\right) \qquad I_2 = a + b\cos\left(\phi + \frac{3}{4}\pi\right) \qquad I_3 = a + b\cos\left(\phi + \frac{5}{4}\pi\right)$$

$$\phi = \tan^{-1}\left(\frac{I_3 - I_2}{I_1 - I_2}\right)$$