Optical Metrology

Lecture 6: Moiré and Triangulation

Sinusoidal gratings

Often physical gratings are square-wave function, but we can describe them by sinusoidal gratings

$$t_1(x, y) = a + a\cos\left(\frac{2\pi}{p}x\right)$$

where p is the grating period and where $0 < a < \frac{1}{2}$.

These gratings become modulated

$$t_2(x, y) = a + a\cos 2\pi\left(\frac{x}{p} + \psi(x)\right) \tag{7.2}$$

$\psi(x)$ is the modulation function and is equal to the displacement of the grating lines from its original position divided by the grating period

$$\psi(x) = \frac{u(x)}{p} \tag{7.3}$$

where $u(x)$ is the displacement.

Sinusoidal gratings

When the two gratings given by Equations (7.1) and (7.2) are laid in contact, the resulting transmittance t becomes the product of the individual transmittances, viz.

$$\begin{aligned} t(x, y) &= t_1 t_2 \\ &= a^2 \left\{ 1 + \cos \frac{2\pi}{p} x + \cos 2\pi \left[\frac{x}{p} + \psi(x) \right] \right. \\ &\quad \left. + \frac{1}{2} \cos 2\pi \left[\frac{2x}{p} + \psi(x) \right] + \frac{1}{2} \cos 2\pi \psi(x) \right\} \end{aligned} \tag{7.4}$$

- The first three terms represent the original gratings, the fourth term the second grating with doubled frequency,
- The fifth term depends on the modulation function only. It is this term which describes the moiré pattern.

Sinusoidal gratings

When the two gratings given by Equations (7.1) and (7.2) are laid in contact, the resulting transmittance t becomes the product of the individual transmittances, viz.

$$\begin{aligned} t(x, y) &= t_1 t_2 \\ &= a^2 \left\{ 1 + \cos \frac{2\pi}{p} x + \cos 2\pi \left[\frac{x}{p} + \psi(x) \right] \right. \\ &\quad \left. + \frac{1}{2} \cos 2\pi \left[\frac{2x}{p} + \psi(x) \right] + \frac{1}{2} \cos 2\pi \psi(x) \right\} \end{aligned} \tag{7.4}$$

a maximum resulting in a bright fringe whenever

$$\psi(x) = n, \quad \text{for} \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{7.6}$$

and minima (dark fringes) whenever

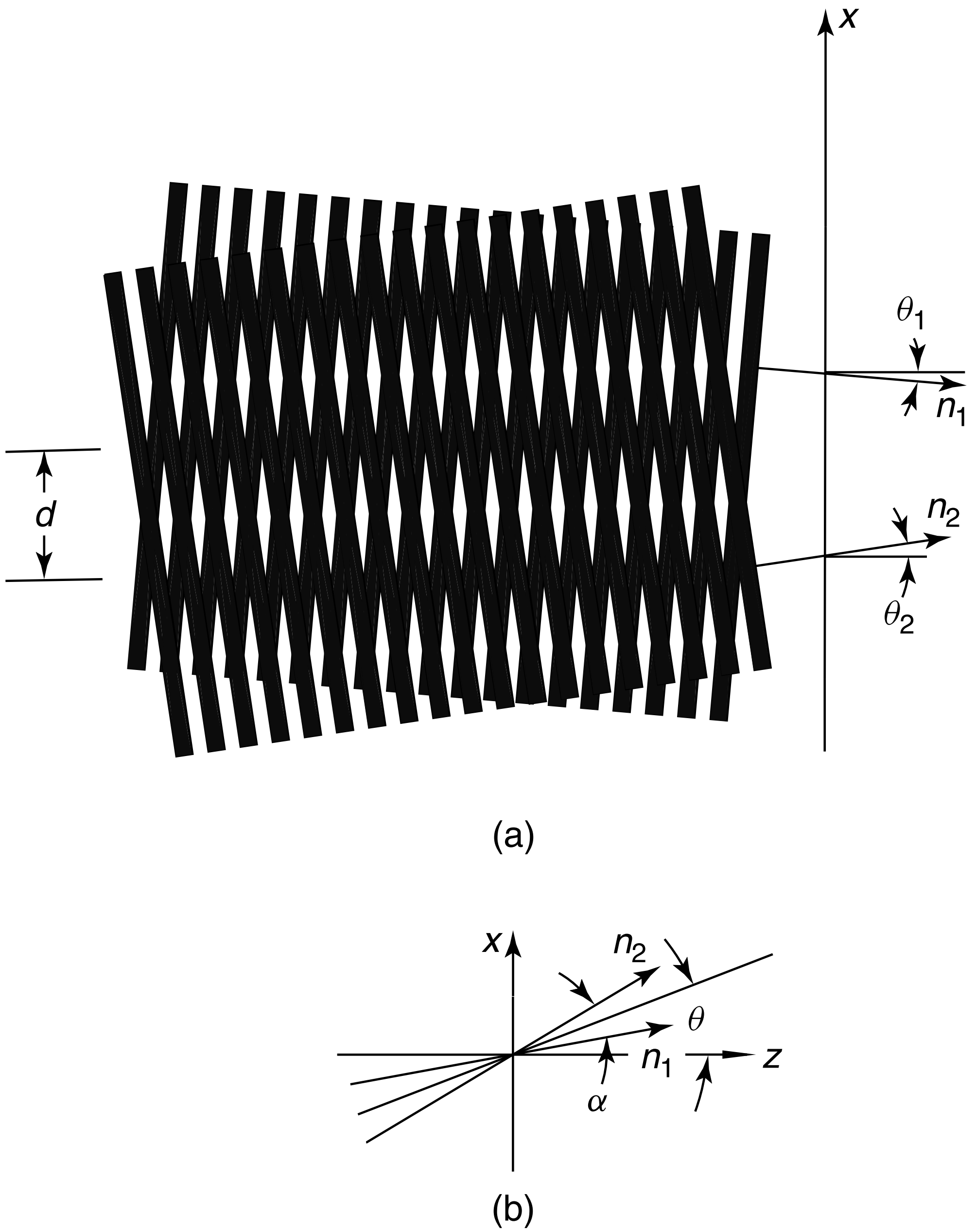
$$\psi(x) = n + \tfrac{1}{2}, \quad \text{for} \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{7.7}$$

Both grating t_1 and t_2 could be phase-modulated by modulation functions ψ_1 and ψ_2 respectively. Then $\psi(x)$ in Equations (7.6) and (7.7) has to be replaced by

$$\psi(x) = \psi_1(x) - \psi_2(x) \tag{7.8}$$

Moiré

- The figure is an illustration of two interfering plane waves.
- Two gratings that lie in contact, with a small angle between the grating lines.
- As a result, we see a fringe pattern of much lower frequency than the individual gratings.
- This is an example of the moiré effect and the resulting fringes are a moiré pattern.



Moiré

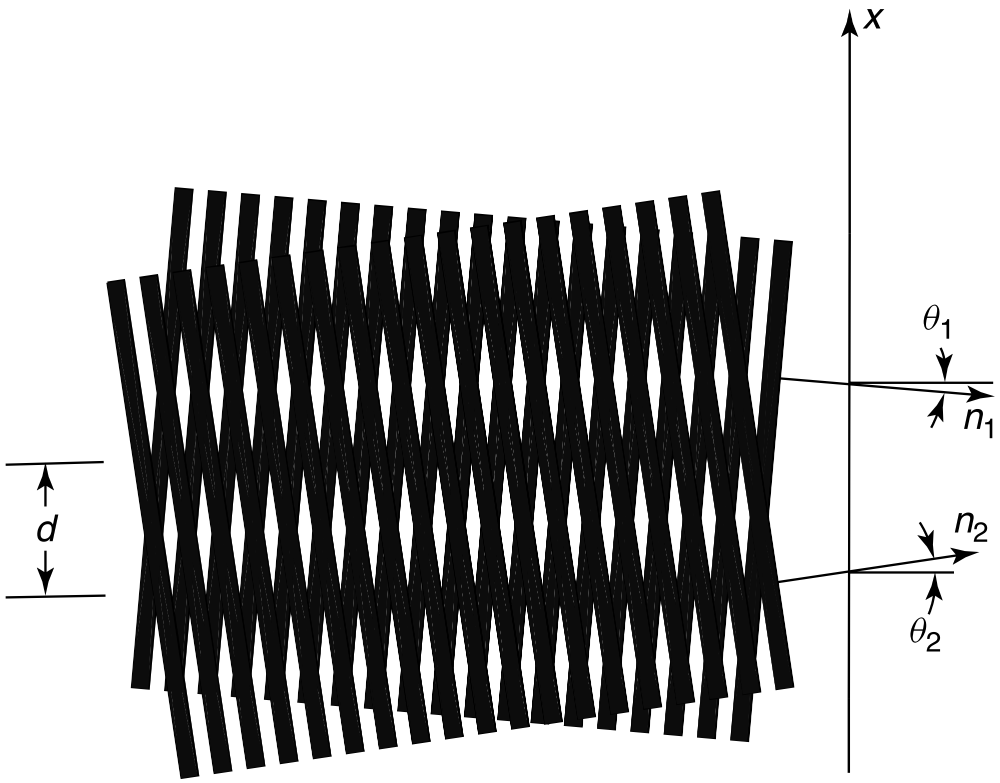
Transmittance is equal to the product

$$t(x, y) = t_1 t_2$$

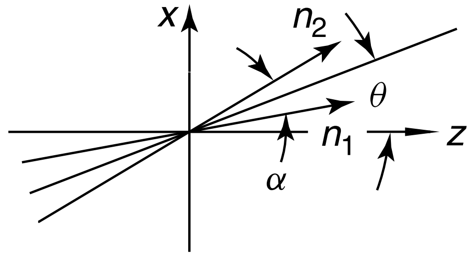
Interfringe distance

$$d = \frac{p}{2 \sin \frac{\alpha}{2}}$$

This can be applied for measuring α by measurement of d.



(a)



(b)

Measurement of In-Plane Deformation and Strains

When measuring in-plane deformations a grating is attached to the test surface. When the surface is deformed, the grating will follow the deformation and will therefore be given by Equation (7.2). The deformation $u(x)$ will be given directly from Equation (7.3):

$$u(x) = p\psi(x) \tag{7.10}$$

To obtain the moiré pattern, one may apply one of several methods (Post 1982; Sciammarella 1972, 1982):

(1) Place the reference grating with transmittance t_1 in contact with the model grating with transmittance t_2. The resulting intensity distribution then becomes proportional to the product $t_1 \cdot t_2$.

Measurement of In-Plane Deformation and Strains

To obtain the moiré pattern, one may apply one of several methods (Post 1982; Sciammarella 1972, 1982):

(1) Place the reference grating with transmittance t_1 in contact with the model grating with transmittance t_2. The resulting intensity distribution then becomes proportional to the product $t_1 \cdot t_2$.

We obtain:

$$I(x) = I_0 + I_1 \cos 2\pi\psi(x) + \text{terms of higher frequencies}$$

A DC term I_0 and a term containing the modulation function.

When applying low frequency gratings -> OK.

High frequency -> Low contrast Moiré fringes.

Measurement of In-Plane Deformation and Strains

We can filter optically or digitally to obtain:

$$\text{maximum whenever } \psi(x) = n, \quad \text{for} \quad n = 0, 1, 2, \ldots$$

$$\text{minimum whenever } \psi(x) = n + \tfrac{1}{2}, \quad \text{for} \quad n = 0, 1, 2, \ldots$$

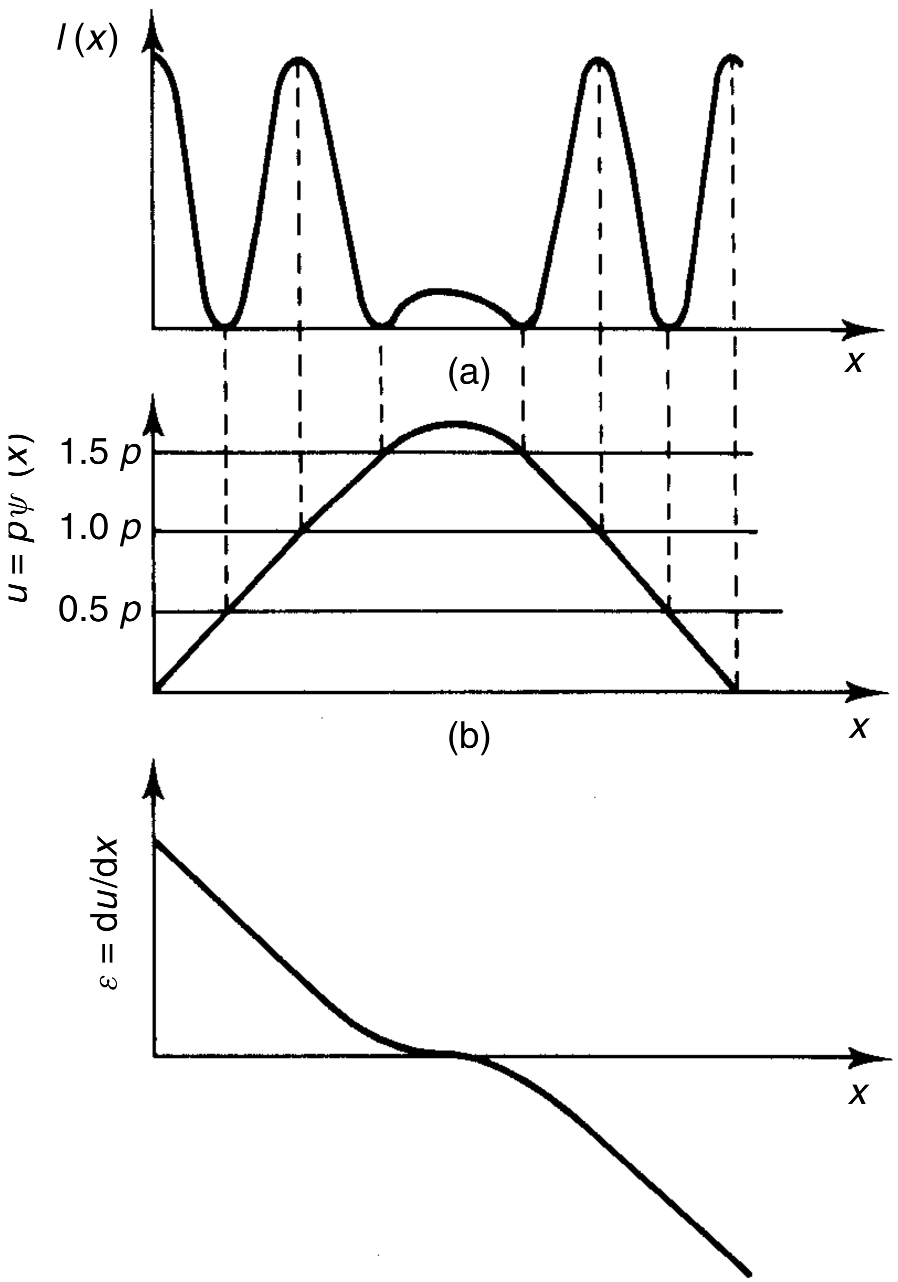
According to Equation (7.10) this corresponds to a displacement equal to

$$u(x) = np \quad \text{for maxima} \tag{7.13a}$$

$$u(x) = (n + \tfrac{1}{2})p \quad \text{for minima} \tag{7.13b}$$

Measurement of In-Plane Deformation and Strains

Intensity distribution of Moiré pattern, displacement and strain.



Measurement of In-Plane Deformation and Strains

By orienting the model grating and the reference grating along the y-axis, we can in the same manner find the modulation function $\psi_y(y)$ and the displacement $v(y)$ in the y-direction. $\psi_x(x)$ and $\psi_y(y)$ can be detected simultaneously by applying crossed gratings, i.e. gratings of orthogonal lines in the x- and y-directions. Thus we also are able to calculate the strains

$$\varepsilon_x = p \frac{\partial \psi_x}{\partial x} \tag{7.14a}$$

$$\varepsilon_y = p \frac{\partial \psi_y}{\partial y} \tag{7.14b}$$

$$\gamma_{xy} = p \left[\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right] \tag{7.14c}$$

Measurement of Out-of-Plane Deformations. Contouring

An effect where moiré fringes are formed between a grating and its own shadow.

Shadow Moiré

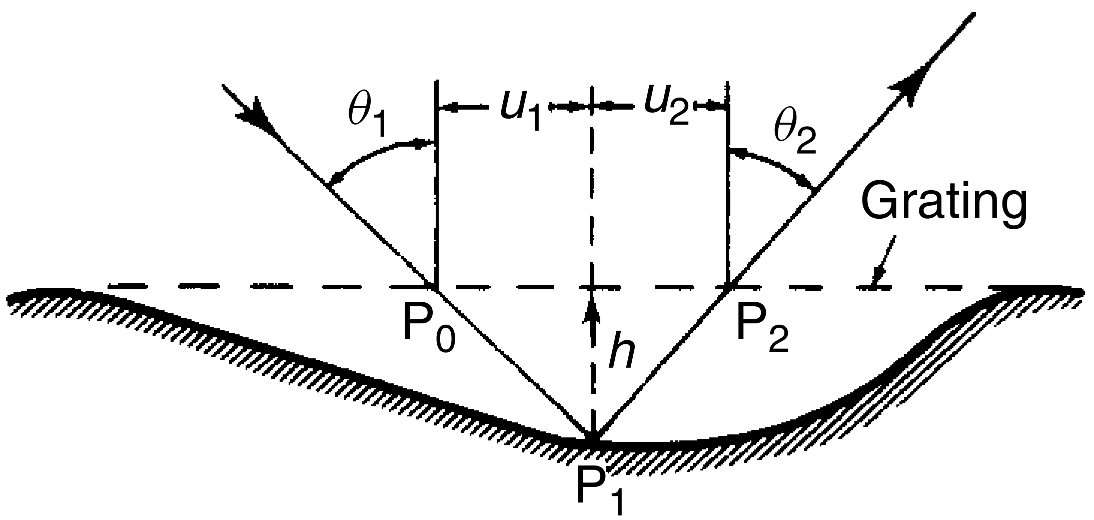


Figure 7.3 Shadow moiré

P0 is projected onto P1, then by viewing is projected to P2 on the grating.

Equivalent to displacement:

$$u = u_1 + u_2 = h(x, y)(\tan\theta_1 + \tan\theta_2)$$

$h(x, y)$ is the height difference

Measurement of Out-of-Plane Deformations. Contouring

This corresponds to a modulation function

$$\psi(x) = \frac{u}{p} = \frac{h(x, y)}{p}(\tan\theta_1 + \tan\theta_2)$$

A bright fringe is obtained when

$\psi(x) = n$, for $n = 0, 1, 2, \ldots,$

$$h(x, y) = \frac{np}{\tan\theta_1 + \tan\theta_2}$$

Dark fringe

$$h(x, y) = \frac{(n + \frac{1}{2})p}{\tan\theta_1 + \tan\theta_2}$$

Shadow Moiré

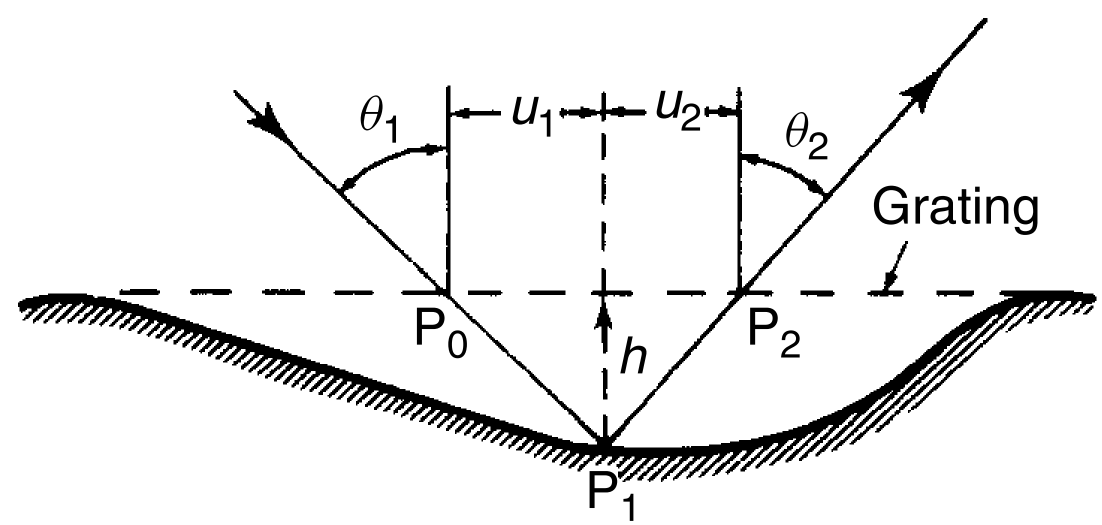


Figure 7.3 Shadow moiré

Triangulation

beam is incident on a diffusely scattering surface under an angle θ_1. The resulting light spot on the surface is imaged by a lens onto a detector D. The optical axis of the lens makes an angle θ_2 to the surface normal. Assume that the object moves a distance s normal to its surface. From the figure, using simple trigonometric relations, we find that

$$s' = m\frac{s\sin(\theta_1+\theta_2)}{\cos\theta_1} = ms(\tan\theta_1\cos\theta_2+\sin\theta_2)$$

where m is the transversal magnification of the lens.

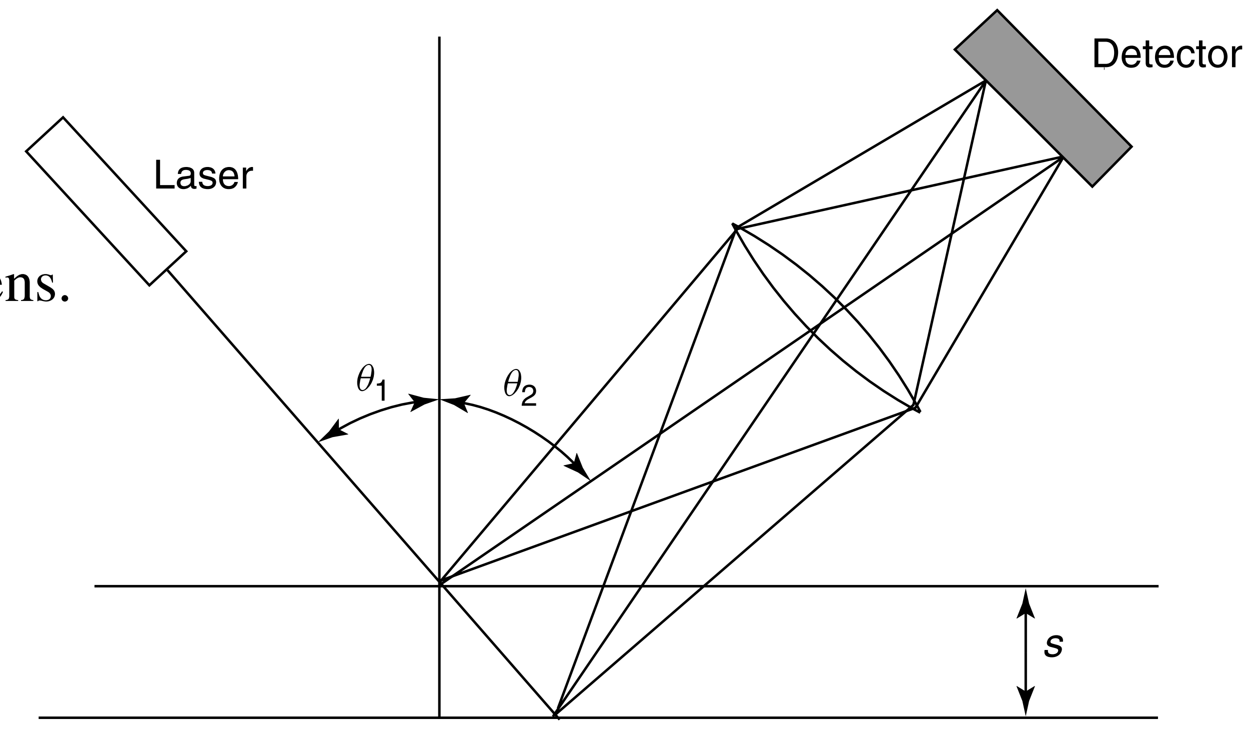


Figure 7.13 Triangulation probe