Optical Metrology. Kjell J. Gåsvik

ISBN: 0-470-84300-4

1

Basics

1.1 INTRODUCTION

Before entering into the different techniques of optical metrology some basic terms and definitions have to be established. Optical metrology is about light and therefore we must develop a mathematical description of waves and wave propagation, introducing important terms like wavelength, phase, phase fronts, rays, etc. The treatment is kept as simple as possible, without going into complicated electromagnetic theory.

1.2 WAVE MOTION. THE ELECTROMAGNETIC SPECTRUM

Figure 1.1 shows a snapshot of a harmonic wave that propagates in the z-direction. The disturbance $\psi(z, t)$ is given as

$$\psi(z, t) = U \cos\left[2\pi\left(\frac{z}{\lambda} - \nu t\right) + \delta\right] \tag{1.1}$$

The argument of the cosine function is termed the phase and δ the phase constant. Other parameters involved are

U = the amplitude
λ = the wavelength
ν = the frequency (the number of waves per unit time)
$k = 2\pi/\lambda$ the wave number

The relation between the frequency and the wavelength is given by

$$\lambda\nu = v \tag{1.2}$$

where

v = the wave velocity

$\psi(z, t)$ might represent the field in an electromagnetic wave for which we have

$$v = c = 3 \times 10^8 \text{ m/s}$$

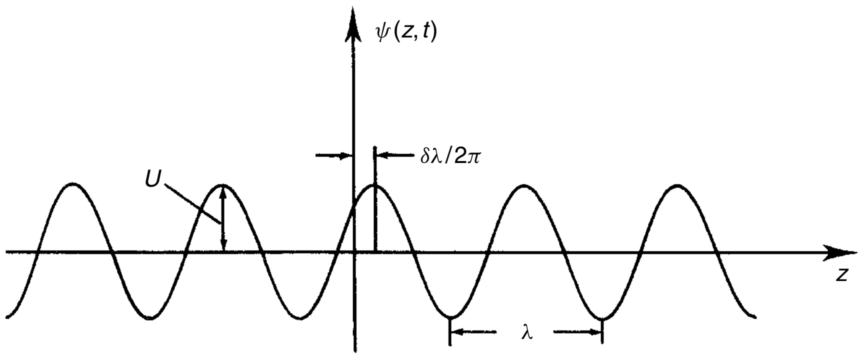
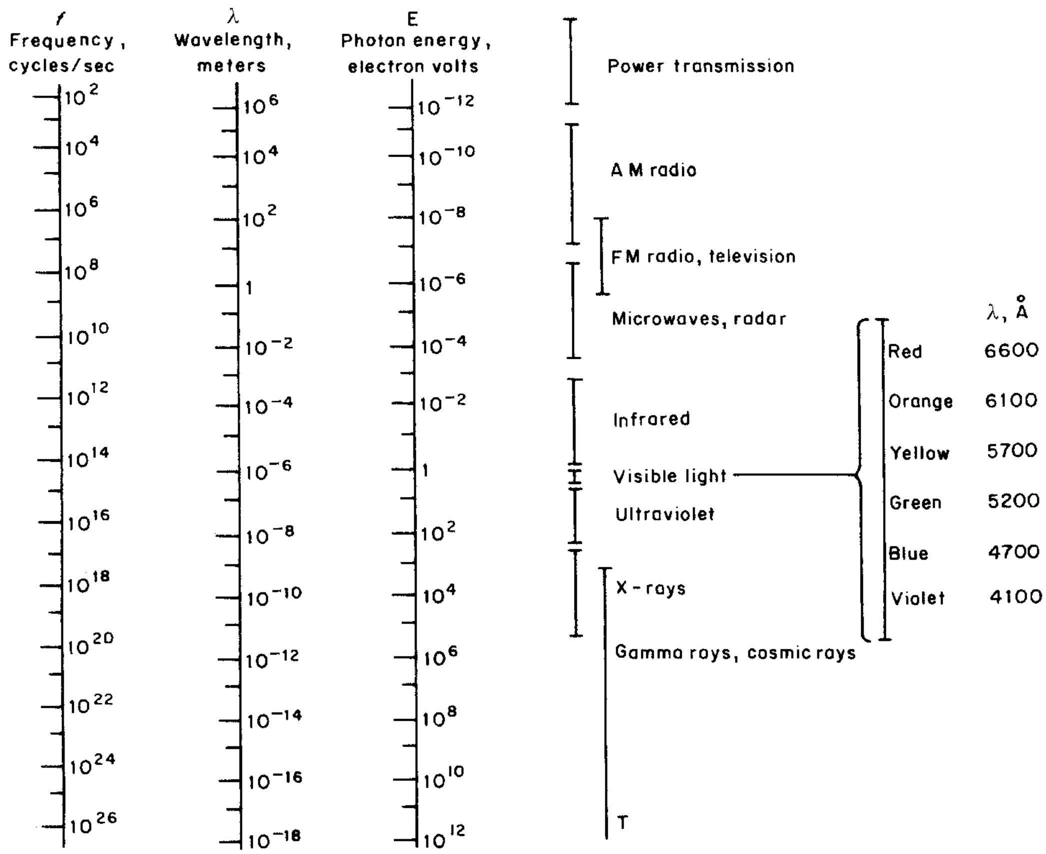


Figure 1.1 Harmonic wave

Table 1.1 The electromagnetic spectrum (From Young (1968))



The ratio of the speed c of an electromagnetic wave in vacuum to the speed v in a medium is known as the absolute index of refraction n of that medium

$$n = \frac{c}{v} \tag{1.3}$$

The electromagnetic spectrum is given in Table 1.1.

Although it does not really affect our argument, we shall mainly be concerned with visible light where

$\lambda = 400–700$ nm (1 nm $= 10^{-9}$ m)
$\nu = (4.3–7.5) \times 10^{14}$ Hz

1.3 THE PLANE WAVE. LIGHT RAYS

Electromagnetic waves are not two dimensional as in Figure 1.1, but rather three-dimensional waves. The simplest example of such waves is given in Figure 1.2 where a plane wave that propagates in the direction of the **k**-vector is sketched. Points of equal phase lie on parallel planes that are perpendicular to the propagation direction. Such planes are called phase planes or phase fronts. In the figure, only some of the infinite number of phase planes are drawn. Ideally, they should also have infinite extent.

Equation (1.1) describes a plane wave that propagates in the z-direction. ($z =$ constant gives equal phase for all x, y, i.e. planes that are normal to the z-direction.) In the general case where a plane wave propagates in the direction of a unit vector $\boldsymbol{n}$, the expression describing the field at an arbitrary point with radius vector $\mathbf{r} = (x, y, z)$ is given by

$$\psi(x, y, z, t) = U\cos[k\mathbf{n} \cdot \mathbf{r} - 2\pi\nu t + \delta] \tag{1.4}$$

That the scalar product fulfilling the condition $\mathbf{n} \cdot \mathbf{r} =$ constant describes a plane which is perpendicular to $\mathbf{n}$ is shown in the two-dimensional case in Figure 1.3. That this is correct also in the three-dimensional case is easily proved.

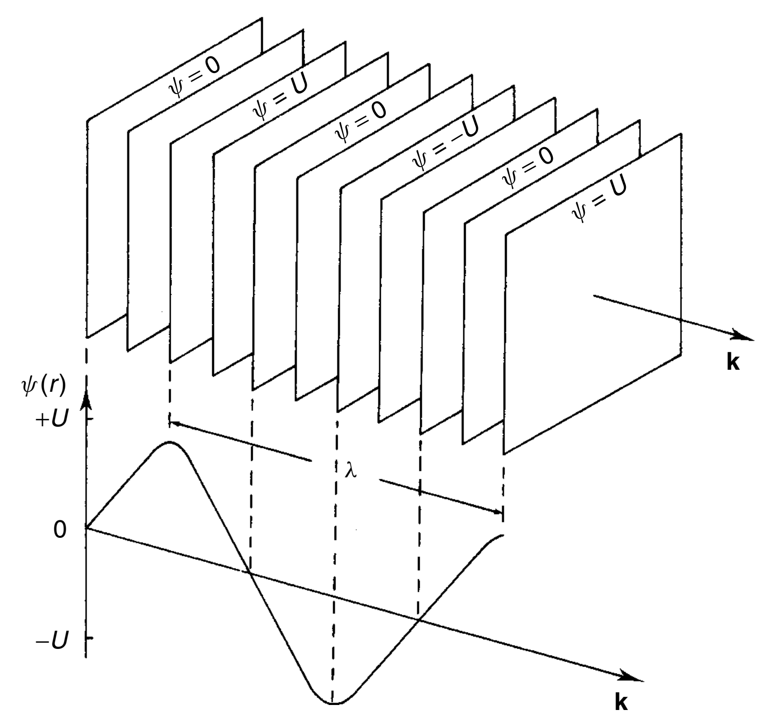


Figure 1.2 The plane wave

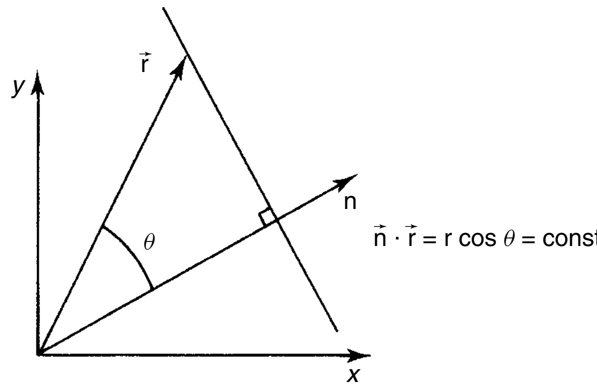


Figure 1.3

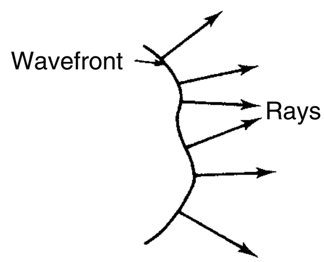


Figure 1.4

Next we give the definition of light rays. They are directed lines that are everywhere perpendicular to the phase planes. This is illustrated in Figure 1.4 where the cross-section of a rather complicated wavefront is sketched and where some of the light rays perpendicular to the wavefront are drawn.

1.4 PHASE DIFFERENCE

Let us for a moment turn back to the plane wave described by Equation (1.1). At two points z_1 and z_2 along the propagation direction, the phases are $\phi_1 = kz_1 - 2\pi\nu t + \delta$ and $\phi_2 = kz_2 - 2\pi\nu t + \delta$ respectively, and the phase difference

$$\Delta\phi = \phi_1 - \phi_2 = k(z_1 - z_2) \tag{1.5}$$

Hence, we see that the phase difference between two points along the propagation direction of a plane wave is equal to the geometrical path-length difference multiplied by the wave number. This is generally true for any light ray. When the light passes a medium different from air (vacuum), we have to multiply by the refractive index n of the medium, such that

$$\text{optical path length} = n \times (\text{geometrical path length})$$

$$\text{phase difference} = k \times (\text{optical path length})$$

1.5 COMPLEX NOTATION. COMPLEX AMPLITUDE

The expression in Equation (1.4) can be written in complex form as

$$\psi(x, y, z, t) = \mathrm{Re}\{U\mathrm{e}^{\mathrm{i}(\phi - 2\pi\nu t)}\} \tag{1.6a}$$

where

$$\phi = k\mathbf{n} \cdot \mathbf{r} + \delta \tag{1.6b}$$

is the spatial dependent phase. In Appendix A, some simple arithmetic rules for complex numbers are given.

In the description of wave phenomena, the notation of Equation (1.6) is commonly adopted and 'Re' is omitted because it is silently understood that the field is described by the real part.

One advantage of such complex representation of the field is that the spatial and temporal parts factorize:

$$\psi(x, y, z, t) = U\mathrm{e}^{\mathrm{i}(\phi - 2\pi\nu t)} = U\mathrm{e}^{\mathrm{i}\phi}\mathrm{e}^{-\mathrm{i}2\pi\nu t} \tag{1.7}$$

In optical metrology (and in other branches of optics) one is most often interested in the spatial distribution of the field. Since the temporal-dependent part is known for each frequency component, we therefore can omit the factor $\mathrm{e}^{-\mathrm{i}2\pi\nu t}$ and only consider the spatial complex amplitude

$$u = U\mathrm{e}^{\mathrm{i}\phi} \tag{1.8}$$

This expression describes not only a plane wave, but a general three-dimensional wave where both the amplitude U and the phase ϕ may be functions of x, y and z.

Figure 1.5(a, b) shows examples of a cylindrical wave and a spherical wave, while in Figure 1.5(c) a more complicated wavefront resulting from reflection from a rough surface is sketched. Note that far away from the point source in Figure 1.5(b), the spherical wave is nearly a plane wave over a small area. A point source at infinity, represents a plane wave.

1.6 OBLIQUE INCIDENCE OF A PLANE WAVE

In optics, one is often interested in the amplitude and phase distribution of a wave over fixed planes in space. Let us consider the simple case sketched in Figure 1.6 where a plane wave falls obliquely on to a plane parallel to the xy-plane a distance z from it. The wave propagates along the unit vector $\mathbf{n}$ which is lying in the xz-plane (defined as the plane of incidence) and makes an angle θ to the z-axis. The components of the $\mathbf{n}$- and $\mathbf{r}$-vectors are therefore

$$\mathbf{n} = (\sin\theta, 0, \cos\theta)$$
$$\mathbf{r} = (x, y, z)$$

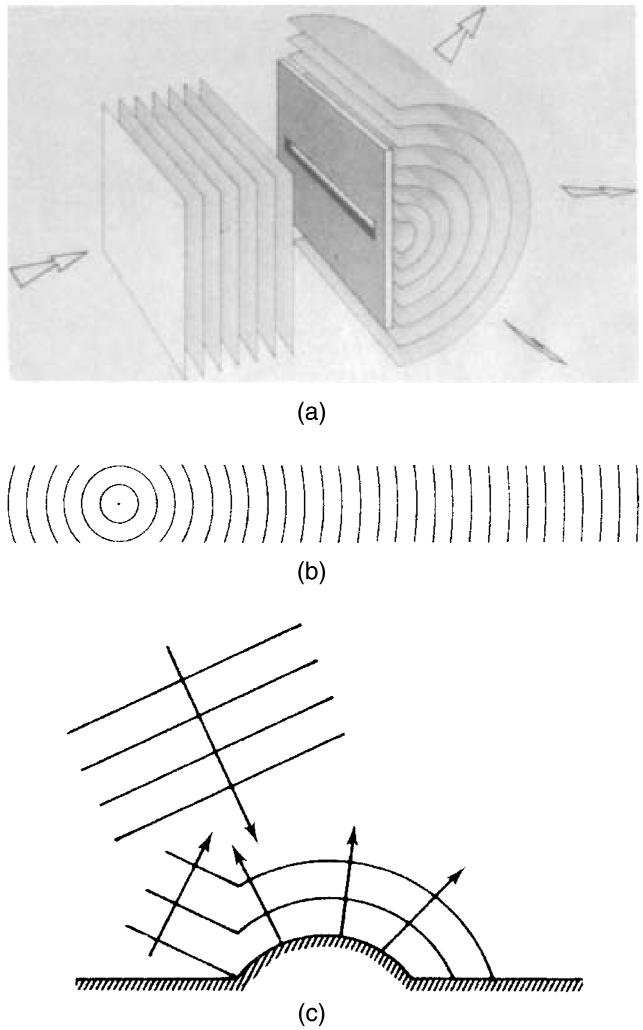


Figure 1.5 ((a) and (b) from Hecht & Zajac (1974), Figures 2.16 and 2.17. Reprinted with permission.)

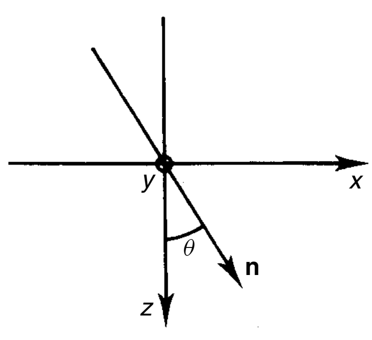


Figure 1.6

These expressions put into Equation (1.6) (Re and temporal part omitted) give

$$u = U\mathrm{e}^{\mathrm{i}k(x\sin\theta + z\cos\theta)} \tag{1.9a}$$

For $z = 0$ (the xy-plane) this reduces to

$$u = U\mathrm{e}^{\mathrm{i}kx\sin\theta} \tag{1.9b}$$

1.7 THE SPHERICAL WAVE

A spherical wave, illustrated in Figure 1.5(b), is a wave emitted by a point source. It should be easily realized that the complex amplitude representing a spherical wave must be of the form

$$u = \frac{U}{r}\mathrm{e}^{\mathrm{i}kr} \tag{1.10}$$

where r is the radial distance from the point source. We see that the phase of this wave is constant for $r =$ constant, i.e. the phase fronts are spheres centred at the point source. The r in the denominator of Equation (1.10) expresses the fact that the amplitude decreases as the inverse of the distance from the point source.

Consider Figure 1.7 where a point source is lying in the x_0, y_0-plane at a point of coordinates x_0, y_0. The field amplitude in a plane parallel to the x_0y_0-plane at a distance z then will be given by Equation (1.10) with

$$r = \sqrt{z^2 + (x - x_0)^2 + (y - y_0)^2} \tag{1.11}$$

where x, y are the coordinates of the illuminated plane. This expression is, however, rather cumbersome to work with. One therefore usually makes some approximations, the first of which is to replace z for r in the denominator of Equation (1.10). This approximation cannot be put into the exponent since the resulting error is multiplied by the very large

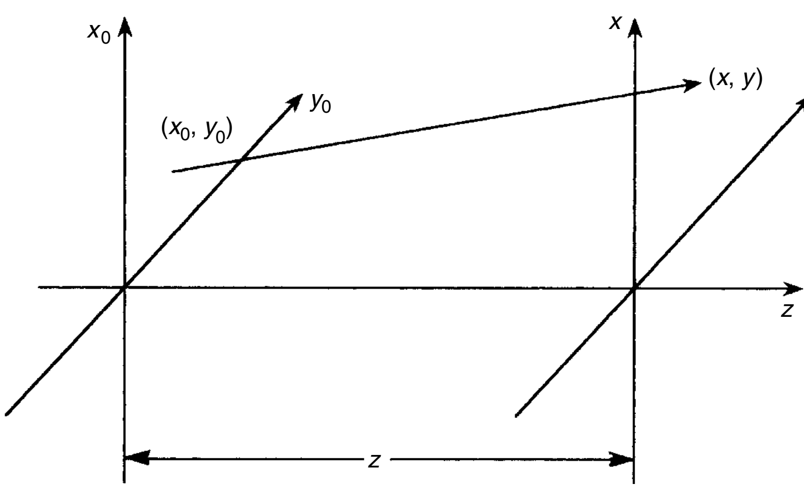


Figure 1.7

number k. A convenient means for approximation of the phase is offered by a binomial expansion of the square root, viz.

$$r = z\sqrt{1 + \left(\frac{x - x_0}{z}\right)^2 + \left(\frac{y - y_0}{z}\right)^2} \approx z\left[1 + \frac{1}{2}\left(\frac{x - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y - y_0}{z}\right)^2\right] \quad (1.12)$$

where r is approximated by the two first terms of the expansion.

The complex field amplitude in the xy-plane resulting from a point source at x_0, y_0 in the $x_0 y_0$-plane is therefore given by

$$u(x, y, z) = \frac{U}{z} e^{ikz} e^{i(k/2z)[(x-x_0)^2+(y-y_0)^2]} \quad (1.13)$$

The approximations leading to this expression are called the Fresnel approximations. We shall here not discuss the detailed conditions for its validity, but it is clear that $(x - x_0)$ and $(y - y_0)$ must be much less than the distance z.

1.8 THE INTENSITY

With regard to the registration of light, we are faced with the fact that media for direct recording of the field amplitude do not exist. The most common detectors (like the eye, photodiodes, multiplication tubes, photographic film, etc.) register the irradiance (i.e. effect per unit area) which is proportional to the field amplitude absolutely squared:

$$I = |u|^2 = U^2 \quad (1.14)$$

This important quantity will hereafter be called the intensity.

We mention that the correct relation between U^2 and the irradiance is given by

$$I = \frac{\varepsilon v}{2} U^2 \quad (1.15)$$

where v is the wave velocity and ε is known as the electric permittivity of the medium. In this book, we will need this relation only when calculating the transmittance at an interface (see Section 9.5).

1.9 GEOMETRICAL OPTICS

For completeness, we refer to the three laws of geometrical optics:

(1) Rectilinear propagation in a uniform, homogeneous medium.

(2) Reflection. On reflection from a mirror, the angle of reflection is equal to the angle of incidence (see Figure 1.8). In this context we mention that on reflection (scattering) from a rough surface (roughness $>\lambda$) the light will be scattered in all directions (see Figure 1.9).

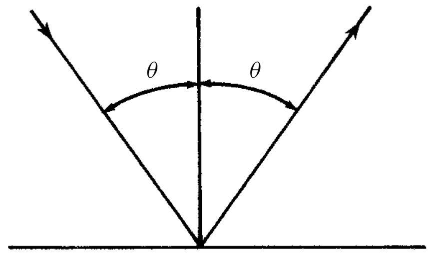


Figure 1.8 The law of reflection

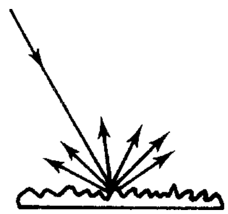


Figure 1.9 Scattering from a rough surface

(3) Refraction. When light propagates from a medium of refractive index n_1 into a medium of refractive index n_2, the propagation direction changes according to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1.16}$$

where θ_1 is the angle of incidence and θ_2 is the angle of emergence (see Figure 1.10). From Equation (1.16) we see that when $n_1 > n_2$, we can have $\theta_2 = \pi/2$. This occurs for an angle of incidence called the critical angle given by

$$\sin\theta_1 = \frac{n_2}{n_1} \tag{1.17}$$

This is called total internal reflection and will be treated in more detail in Section 9.5.

Finally, we also mention that for light reflected at the interface in Figure 1.10, when $n_1 < n_2$, the phase is changed by π.

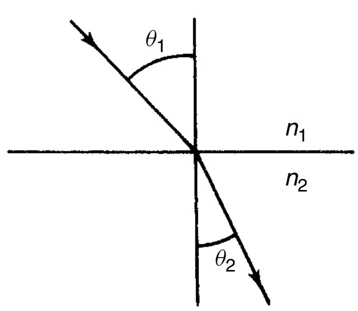


Figure 1.10 The law of refraction

1.10 THE SIMPLE CONVEX (POSITIVE) LENS

We shall here not go into the general theory of lenses, but just mention some of the more important properties of a simple, convex, ideal lens. For more details, see Chapter 2 and Section 4.6.

Figure 1.11 illustrates the imaging property of the lens. From an object point P_o, light rays are emitted in all directions. That this point is imaged means that all rays from P_o which pass the lens aperture D intersect at an image point P_i.

To find P_i, it is sufficient to trace just two of these rays. Figure 1.12 shows three of them. The distance b from the lens to the image plane is given by the lens formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{1.18}$$

and the transversal magnification

$$m = \frac{h_i}{h_o} = \frac{b}{a} \tag{1.19}$$

In Figure 1.13(a), the case of a point source lying on the optical axis forming a spherical diverging wave that is converted to a converging wave and focuses onto a point on the optical axis is illustrated. In Figure 1.13(b) the point source is lying on-axis at a distance

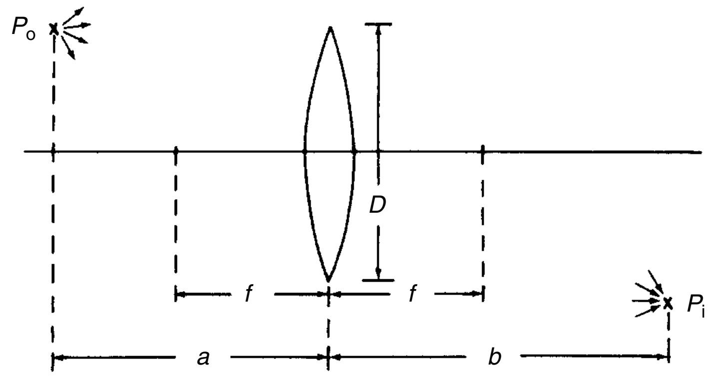


Figure 1.11

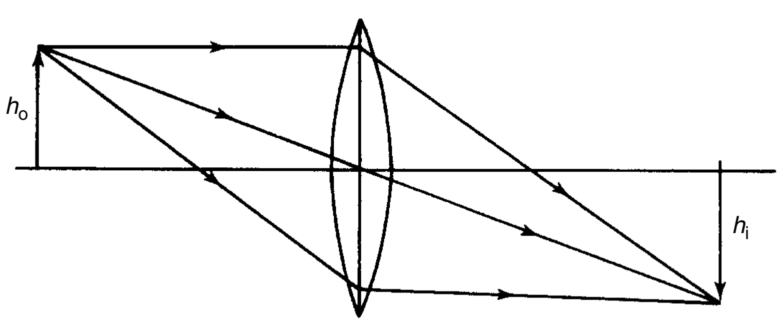


Figure 1.12

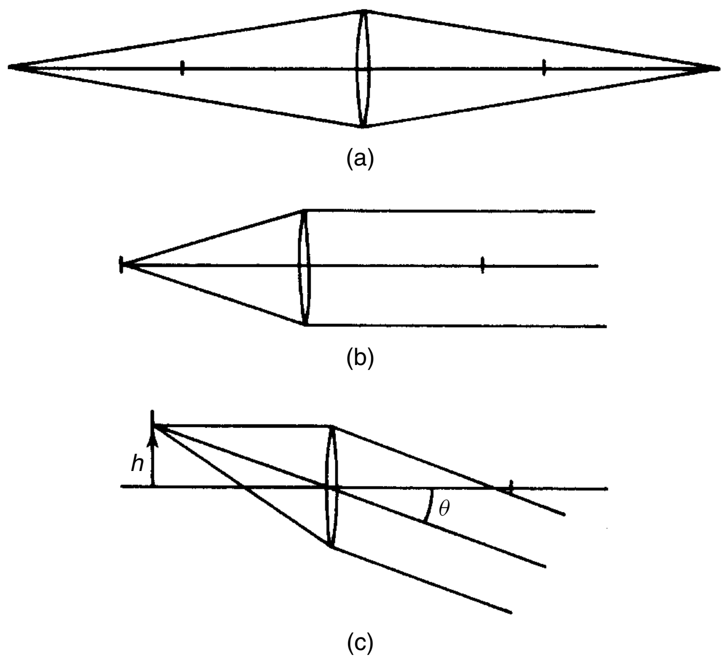


Figure 1.13

from the lens equal to the focal length f. We then get a plane wave that propagates along the optical axis. In Figure 1.13(c) the point source is displaced along the focal plane a distance h from the optical axis. We then get a plane wave propagating in a direction that makes an angle θ to the optical axis where

$$\tan\theta = h/f \tag{1.20}$$

1.11 A PLANE-WAVE SET-UP

Finally, we refer to Figure 1.14 which shows a commonly applied set-up to form a uniform, expanded plane wave from a laser beam. The laser beam is a plane wave with a small cross-section, typically 1 mm. To increase the cross-section, the beam is first directed through lens L_1, usually a microscope objective which is a lens of very short focal length f_1. A lens L_2 of greater diameter and longer focal length f_2 is placed as shown in the figure. In the focal point of L_1 a small opening (a pinhole) of diameter typically 10 μm is placed. In that way, light which does not fall at the focal point is blocked. Such stray light is due to dust and impurities crossed by the laser beam on its

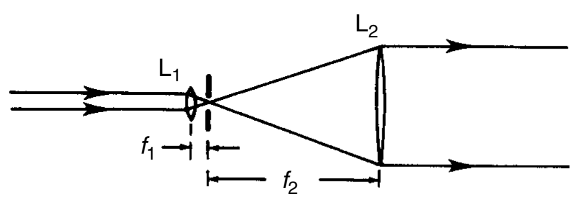


Figure 1.14 A plane wave set-up

way via other optical elements (like mirrors, beamsplitters, etc.) and it causes the beam not to be a perfect plane wave.

PROBLEMS

1.1 How many 'yellow' light waves ($\lambda = 550$ nm) will fit into a distance in space equal to the thickness of a piece of paper (0.1 mm)? How far will the same number of microwaves ($\nu = 10^{10}$ Hz, i.e 10 GHz, and $v = 3 \times 10^8$ m/s) extend?

1.2 Using the wave functions

$$\psi_1 = 4 \sin 2\pi(0.2z - 3t)$$
$$\psi_2 = \frac{\sin(7z + 3.5t)}{2.5}$$

determine in each case (a) the frequency, (b) wavelength, (c) period, (d) amplitude, (e) phase velocity and (f) direction of motion. Time is in seconds and z in metres.

1.3 Consider the plane electromagnetic wave (in SI units) given by the expressions $U_x = 0$, $U_y = \exp i[2\pi \times 10^{14}(t - x/c) + \pi/2]$, and $U_z = 0$.

What is the frequency, wavelength, direction of propagation, amplitude and phase constant of the wave?

1.4 A plane, harmonic light wave has an electric field given by

$$U_z = U_0 \exp i\left[\pi 10^{15}\left(t - \frac{x}{0.65c}\right)\right]$$

while travelling in a piece of glass. Find

(a) the frequency of the light,

(b) its wavelength,

(c) the index of refraction of the glass.

1.5 Imagine that we have a non-absorbing glass plate of index n and thickness Δz which stands between a source and an observer.

(a) If the unobstructed wave (without the plate present) is $U_u = U_0 \exp i\omega(t - z/c)$, ($\omega = 2\pi\nu$) show that with the plate in place the observer sees a wave

$$U_\mathrm{p} = U_0 \exp i\omega\left[t - \frac{(n-1)\Delta z}{c} - \frac{z}{c}\right]$$

(b) Show that if either $n \approx 1$ or Δz is very small, then

$$U_\mathrm{p} = U_\mathrm{u} + \frac{\omega(n-1)\Delta z}{c} U_\mathrm{u} e^{-i\pi/2}$$

The second term on the right may be interpreted as the field arising from the oscillating molecules in the glass plate.

1.6 Show that the optical path, defined as the sum of the products of the various indices times the thicknesses of media traversed by a beam, that is, $\sum_i n_i x_i$, is equivalent to the length of the path in vacuum which would take the same time for that beam to travel.

1.7 Write down an equation describing a sinusoidal plane wave in three dimensions with wavelength λ, velocity v, propagating in the following directions:

(a) $+z$-axis

(b) Along the line $x = y$, $z = 0$

(c) Perpendicular to the planes $x + y + z = \text{const.}$

1.8 Show that the rays from a point source S that are reflected by a plane mirror appear to be coming from the image point S′. Locate S′.

1.9 Consider Figure P1.1. Calculate the deviation Δ produced by the plane parallel slab as a function of n_1, n_2, t, θ.

1.10 The deviation angle δ gives the total deviation of a ray incident onto a prism, see Figure P1.2. It is given by $\delta = \delta_1 + \delta_2$. Minimum deviation occurs when $\delta_1 = \delta_2$.

(a) Show that in this case δ_m, the value of δ, obeys the equation

$$\frac{n_2}{n_1} = \frac{\sin \frac{1}{2}(\alpha + \delta_m)}{\sin \frac{1}{2}\alpha}$$

(b) Find δ_m for $\alpha = 60^\circ$ and $n_2/n_1 = 1.69$.

1.11 (a) Starting with Snell's law prove that the vector refraction equation has the form

$$n_2\mathbf{k}_2 - n_1\mathbf{k}_1 = (n_2 \cos\theta_2 - n_1 \cos\theta_1)\mathbf{u}_n$$

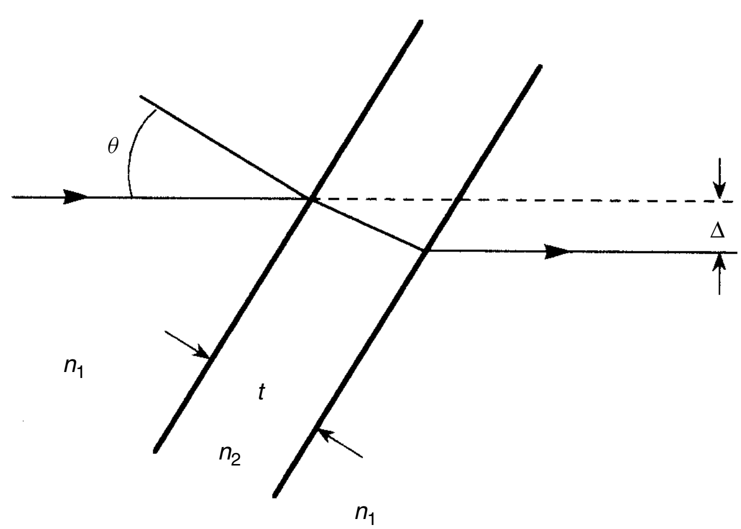


Figure P1.1

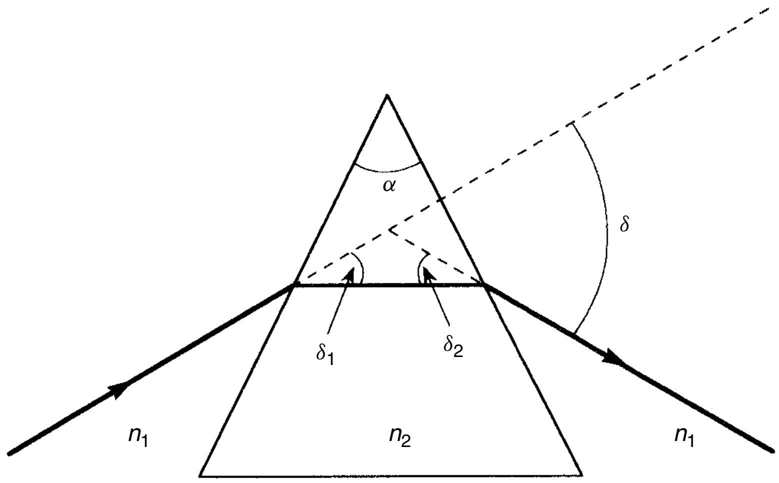


Figure P1.2

where $\mathbf{k}_1$, $\mathbf{k}_2$ are unit propagation vectors and $\mathbf{u}_n$ is the surface normal pointing from the incident to the transmitting medium.

(b) In the same way, derive a vector expression equivalent to the law of reflection.

Optical Metrology. Kjell J. Gåsvik
Copyright © 2002 John Wiley & Sons, Ltd.
ISBN: 0-470-84300-4

2
Gaussian Optics

2.1 INTRODUCTION

Lenses are an important part of most optical systems. Good results in optical measurements often rely on the best selection of lenses. In this chapter we develop the relations governing the passage of light rays through imaging elements on the basis of the paraxial approximation using matrix algebra. We also mention the aberrations occurring when rays deviate from this ideal Gaussian behaviour. Finally we go through some of the standard imaging systems.

2.2 REFRACTION AT A SPHERICAL SURFACE

Consider Figure 2.1 where we have a sphere of radius R centred at C and with refractive index n'. The sphere is surrounded by a medium of refractive index n. A light ray making an angle α with the z-axis is incident on the sphere at a point A at height x above the z-axis. The ray is incident on a plane which is normal to the radius R and the angle of incidence θ is the angle between the ray and the radius from C. The angle of refraction is θ' and the refracted ray is making an angle α' with the z-axis. By introducing the auxiliary angle ϕ we have the following relations:

$$\phi = \theta' - \alpha' \tag{2.1a}$$

$$\phi = \theta - \alpha \tag{2.1b}$$

$$\sin\phi = \frac{x}{R} \tag{2.1c}$$

$$n\sin\theta = n'\sin\theta' \tag{2.1d}$$

The last equation follows from Snell's law of refraction. By assuming the angles to be small we have $\sin\phi \approx \phi$, $\sin\theta \approx \theta$, $\sin\theta' \approx \theta'$ and by combining Equations (2.1) we get the relation

$$\alpha' = \frac{n - n'}{n'R}x + \frac{n}{n'}\alpha = -\frac{P}{n'}x + \frac{n}{n'}\alpha \tag{2.2}$$

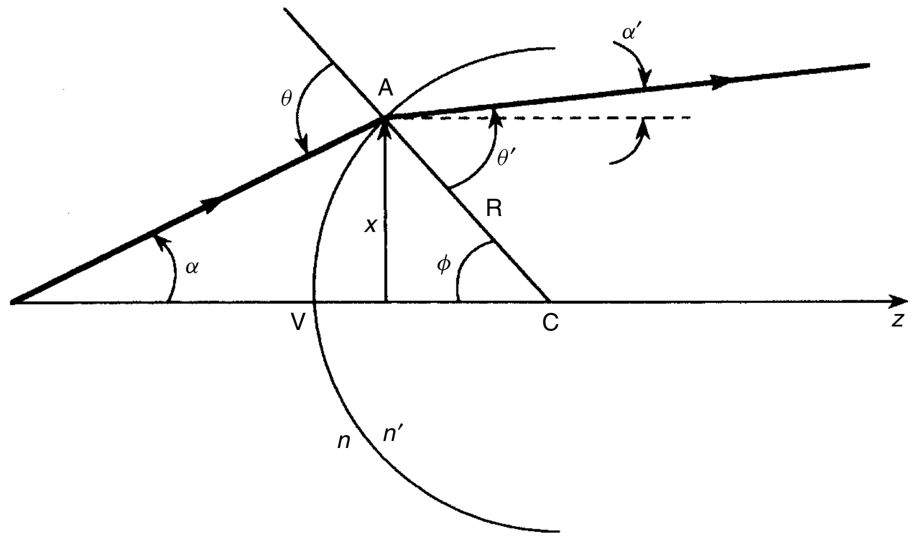


Figure 2.1 Refraction at a spherical interface

where

$$P = \frac{n' - n}{R} \tag{2.3}$$

is called the power of the surface.

The spherical surface in Figure 2.1 might be the front surface of a spherical lens. In tracing rays through optical systems it is important to maintain consistent sign conventions. It is common to define ray angles as positive counterclockwise from the z-axis and negative in the opposite direction. It is also common to define R as positive when the vertex V of the surface is to the left of the centre C and negative when it is to the right of C.

As can be realized, a ray is completely determined at any plane normal to the z-axis by specifying x, its height above the z-axis in that plane, and its angle α relative to the z-axis. A ray therefore can be specified by a column matrix

$$\begin{pmatrix} x \\ \alpha \end{pmatrix}$$

The two components of this matrix will be altered as the ray propagates through an optical system. At the point A in Figure 2.1 the height is unaltered, and this fact can be expressed as

$$x' = x \tag{2.4}$$

The transformation at this point can therefore be expressed in matrix form as

$$\begin{pmatrix} x' \\ \alpha' \end{pmatrix} = R \begin{pmatrix} x \\ \alpha \end{pmatrix} \tag{2.5}$$

where

$$R = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \tag{2.6}$$

is the refraction matrix for the surface.

At this point it is appropriate to point out the approximations involved in reaching this formula. First, we have assumed the ray to lie in the xz-plane. To be general we should have considered the ray to lie in an arbitrary plane, taken its components in the xz- and yz-planes and introduced the component angles α and β relative to the z-axis. We then would have found that x and α at a given point depend only on x and α at other points, not on y and β. In other words, the pairs of variables (x, α) and (y, β) are decoupled from one another and may be treated independently. This is true only within the assumption of small angles. Because of this independence it is not necessary to perform calculations on both projections simultaneously. We do the calculations on the projection in the xz-plane and the answers will also apply for the yz-plane with the substitutions $x \to y$ and $\alpha \to \beta$. The xz projections behave as though y and β were zero. Such rays, which lie in a single plane containing the z-axis are called meridional rays.

In this theory we have assumed that an optical axis can be defined and that all light rays and all normals to refracting or reflecting surfaces make small angles with the axis. Such light rays are called paraxial rays. This first-order approximation was first formulated by C. F. Gauss and is therefore often termed Gaussian optics.

After these remarks we proceed by considering the system in Figure 2.2 consisting of two refracting surfaces with radii of curvature R_1 and R_2 separated by a distance D_{12}. The transformation at the first surface can be written as

$$\begin{pmatrix} x_1' \\ \alpha_1' \end{pmatrix} = R_1 \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} \quad \text{with} \quad R_1 = \begin{pmatrix} 1 & 0 \\ -\frac{P_1}{n_1'} & \frac{n_1}{n_1'} \end{pmatrix} \tag{2.7}$$

where

$$P_1 = \frac{n_1' - n_1}{R_1} \tag{2.8}$$

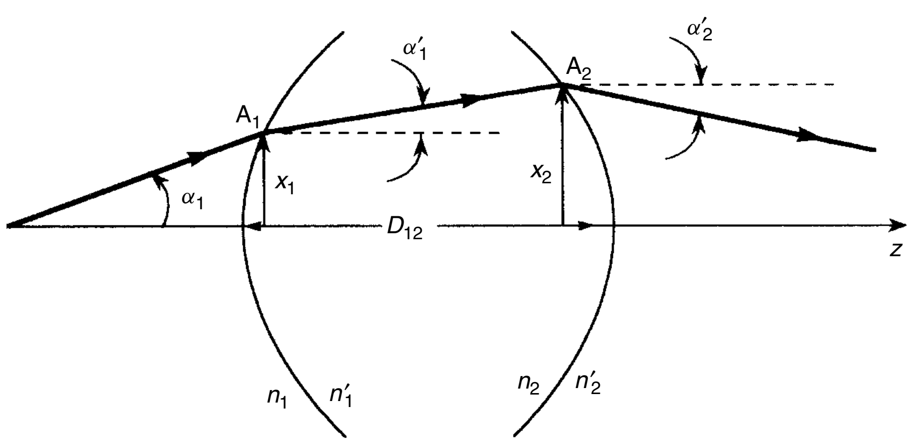


Figure 2.2 Ray tracing through a spherical lens

The translation from A_1 to A_2 is given by

$$x_2 = x'_1 + D_{12}\alpha'_1 \tag{2.9a}$$

$$\alpha_2 = \alpha'_1 \tag{2.9b}$$

which can be written in matrix form as

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = T_{12} \begin{pmatrix} x'_1 \\ \alpha'_1 \end{pmatrix} \quad \text{with} \quad T_{12} = \begin{pmatrix} 1 & D_{12} \\ 0 & 1 \end{pmatrix} \tag{2.10}$$

The refraction at A_2 is described by

$$\begin{pmatrix} x'_2 \\ \alpha'_2 \end{pmatrix} = R_2 \begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} \quad \text{with} \quad R_2 = \begin{pmatrix} 1 & 0 \\ -\dfrac{P_2}{n'_2} & \dfrac{n_2}{n'_2} \end{pmatrix} \tag{2.11}$$

where

$$P_2 = \frac{n'_2 - n_2}{R_2} \tag{2.12}$$

These equations may be combined to give the overall transformation from a point just to the left of A_1 to a point just to the right of A_2:

$$\begin{pmatrix} x'_2 \\ \alpha'_2 \end{pmatrix} = M_{12} \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix} \quad \text{with} \quad M_{12} = R_2 T_{12} R_1 \tag{2.13}$$

This process can be repeated as often as necessary. The linear transformation between the initial position and angle x, α and the final position and angle x', α' can then be written in the matrix form

$$\begin{pmatrix} x' \\ \alpha' \end{pmatrix} = M \begin{pmatrix} x \\ \alpha \end{pmatrix} \tag{2.14}$$

where M is the product of all the refraction and translation matrices written in order, from right to left, in the same sequence followed by the light ray.

The determinant of M is the product of all the determinants of the refraction and translation matrices. We see from Equation (2.10) that the determinant of a translation matrix is always unity and from Equation (2.6) that the determinant of a refraction matrix is given by the ratio of initial to final refractive indices. Thus the determinant of M is the product of the determinants of the separate refraction matrices and takes the form

$$\det\ M = \left(\frac{n_1}{n'_1}\right)\left(\frac{n_2}{n'_2}\right)\cdots \tag{2.15}$$

But $n'_1 = n_2$, $n'_2 = n_3$ and so on, leaving us with

$$\det\ M = \frac{n}{n'} \tag{2.16}$$

where n is the index of the medium to the left of the first refracting surface, and n' is the index of the medium to the right of the last refracting surface.

2.2.1 Examples

(1) *Simple lens*. The matrix M is the same as M_{12} in Equation (2.13). By performing the matrix multiplication using $n_1' = n_2$, $n_1 = n$, $n_2' = n'$ and $D_{12} = d$, we get

$$M = \begin{pmatrix} 1 - \frac{P_1 d}{n_2} & \frac{nd}{n_2} \\ -\frac{P_2}{n'} + \frac{P_1 P_2 d}{n' n_2} - \frac{P_1}{n'} & \frac{n}{n'}\left(1 - \frac{P_2 d}{n_2}\right) \end{pmatrix} \tag{2.17}$$

(2) *Thin lens*. A thin lens is a simple lens with a negligible thickness d. If we let $d \to 0$ (i.e $d \ll R$) in Equation (2.17) we obtain

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \tag{2.18}$$

where the total power is given by (remember the sign convention for R)

$$P = P_1 + P_2 = \frac{n_2 - n}{R_1} + \frac{n' - n_2}{R_2} \tag{2.19}$$

Note that M has the same form for a thin lens as for a single refracting surface. Note also that the matrix elements $M_{11} = 1$ and $M_{12} = 0$. This means that we have $x' = x$, independently of the value of α.

2.3 THE GENERAL IMAGE-FORMING SYSTEM

In a general image-forming system (possibly consisting of several lens elements) an incoming ray at point B is outgoing from point B′, shown schematically in Figure 2.3. The transformation matrix from B to B′ is

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{2.20}$$

where the only requirement so far is

$$\det\ M = \frac{n}{n'} \tag{2.21}$$

We now ask if it is possible to find new reference planes instead of B and B′ for which the general matrix M will take the form of that for a thin lens. These will turn out to

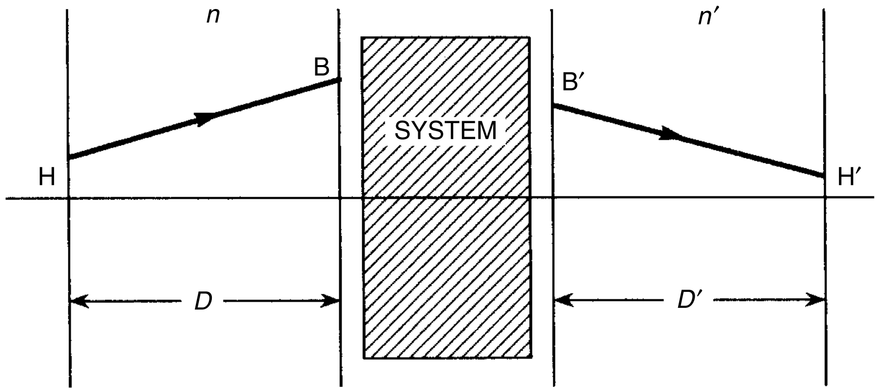


Figure 2.3

be the so-called principal planes and intersect the axis at H and H′ in Figure 2.3. The transformation matrix from the H-plane to the H′-plane can be written in terms of M by adding translation T and T′:

$$M_{\mathrm{HH'}} = T'MT = \begin{pmatrix} 1 & D' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} + D'M_{21} & M_{11}D + M_{12} + D'(M_{21}D + M_{22}) \\ M_{21} & M_{21}D + M_{22} \end{pmatrix} \tag{2.22}$$

The principal planes are defined as planes of unit magnification. Pairs of points in these planes are images of each other and planes with this property are called conjugate planes. Because of this requirement, the 1, 1 element of $M_{\mathrm{HH'}}$ must be unity and the 1, 2 element must be zero, giving

$$M_{\mathrm{HH'}} = \begin{pmatrix} 1 & 0 \\ M_{21} & \dfrac{n}{n'} \end{pmatrix} \tag{2.23}$$

We now equate the elements of the matrices in Equation (2.22) and (2.23)

$$11 : M_{11} + D'M_{21} = 1 \quad \text{i.e.} \quad D' = \frac{1 - M_{11}}{M_{21}} \tag{2.24a}$$

$$22 : M_{21}D + M_{22} = \frac{n}{n'} \quad \text{i.e.} \quad D = \frac{(n/n') - M_{22}}{M_{21}} \tag{2.24b}$$

These equations are meaningful only if the condition

$$M_{21} \neq 0 \tag{2.25}$$

is satisfied. This then becomes the requirement that our general Gaussian system be image-forming. (Identification of matrix element 12 gives the same condition.) To complete the final equivalence between our general image-forming system and a thin lens, it is only necessary to make the identification

$$-\frac{P}{n'} = M_{21} \tag{2.26}$$

Thus the image-formation condition, Equation (2.25) guarantees that our system has non-zero power. This means that all image forming systems have the same formal behaviour in Gaussian optics, as far as ray-tracing is concerned. It should be noted that for an afocal system like the plane wave set-up in Figure 1.14 where the two focal points coincide, $M_{21} = 0$. This is the same configuration as in a telescope where we only have angular magnification.

2.4 THE IMAGE-FORMATION PROCESS

We now want to move from the principal planes to other conjugate planes and determine the object-image relationships that result. This is done by translation transformations over the distances a and b in Figure 2.4. The overall transformation matrix from A to A′ is given by

$$M_{AA'} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{bP}{n'} & a - \frac{abP}{n'} + \frac{nb}{n'} \\ -\frac{P}{n'} & -\frac{aP}{n'} + \frac{n}{n'} \end{pmatrix} \tag{2.27}$$

The image-formation condition is that the 1, 2 element of this matrix be zero:

$$a - \frac{abP}{n'} + \frac{nb}{n'} = 0 \tag{2.28}$$

that is

$$\frac{n}{a} + \frac{n'}{b} = P \tag{2.29}$$

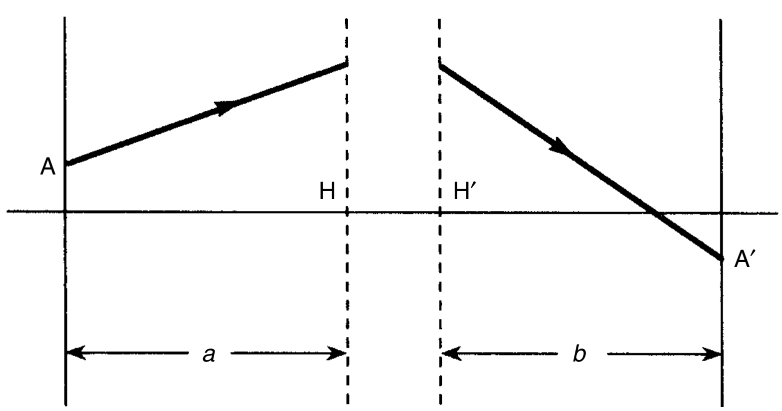


Figure 2.4

When the image is at $+\infty$, the object is in the first focal plane at a distance

$$a = \frac{n}{P} \equiv f \tag{2.30}$$

to the left of the first principal plane. When the object is at $+\infty$, the image is in the second focal plane at a distance

$$b = \frac{n'}{P} \equiv f' \tag{2.31}$$

to the right of the second principal plane. Thus Equation (2.29) may be written in the Gaussian form

$$\frac{n}{a} + \frac{n'}{b} = \frac{n}{f} = \frac{n'}{f'} \tag{2.32}$$

When the refractive indices in image and object space are the same ($n = n'$), this equation takes on the well known form

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{2.33}$$

i.e. the lens formula.

When we have image formation, our matrix can be written

$$M_{\mathrm{AA'}} = \begin{pmatrix} m_x & 0 \\ -\dfrac{P}{n'} & m_\alpha \end{pmatrix} \tag{2.34}$$

where the lateral magnification is

$$m_x = 1 - \frac{bP}{n'} = 1 - \frac{b}{f'} = -\frac{nb}{n'a} \tag{2.35}$$

and the ray angle magnification is

$$m_\alpha = -\frac{aP}{n'} + \frac{n}{n'} = -\frac{a}{b} \tag{2.36}$$

From the condition det $M_{\mathrm{AA'}} = n/n'$ we obtain the result

$$m_x m_\alpha = \frac{n}{n'} \tag{2.37}$$

In addition to the lateral (or transversal) magnification m_x, one might introduce a longitudinal (or axial) magnification defined as $\Delta b/\Delta a$. By differentiating the lens formula, we get $-\Delta a/a^2 - \Delta b/b^2 = 0$, which gives

$$\frac{\Delta b}{\Delta a} = -\left(\frac{b}{a}\right)^2 = -m_x^2 \tag{2.38}$$

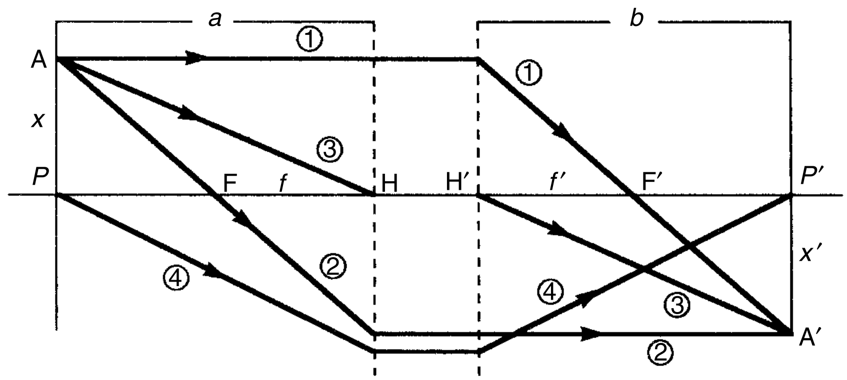


Figure 2.5 Principal planes with some key rays

It should be emphasized that the physical location of the principal planes could be inside one of the components of the image-forming system. Or they could be outside. The point to be made is that these are mathematical planes, and the rays behave as though they were deviated as shown in Figure 2.5. There is no *a priori* reason for the order of the principal planes. The plane H could be to the right of H′. The plane H will be to the right of F and H′ to the left of F′ if f and f' are positive.

2.5 REFLECTION AT A SPHERICAL SURFACE

Spherical mirrors are used as elements in some optical systems. In this section we therefore develop transformations at a reflecting spherical surface.

In Figure 2.6 a light ray making an angle α with the z-axis is incident on the sphere at a point A at height x and is reflected at an angle α' to the z-axis. The sphere centre is

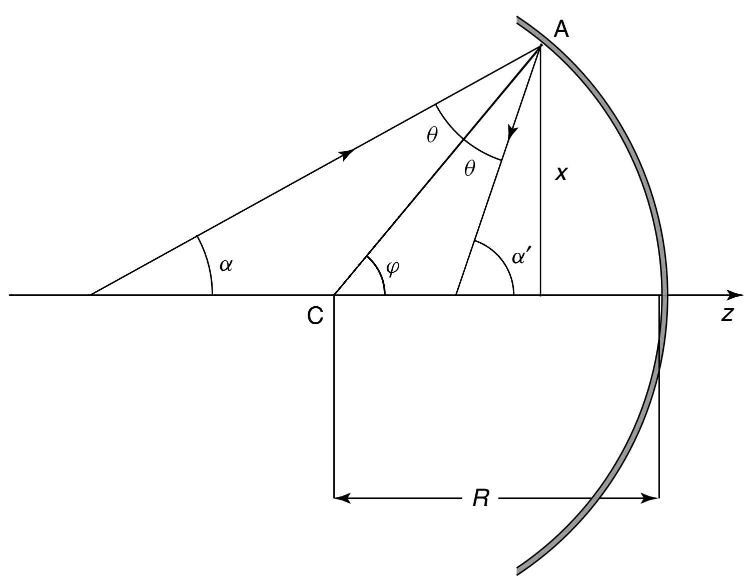


Figure 2.6 Reflection at a spherical surface

at C and therefore the reflection angle θ, equal to the angle of incidence, is as shown in the figure. From the geometry we see that

$$\alpha' = \phi + \theta$$
$$\phi = \alpha + \theta$$

which gives

$$\alpha' = 2\phi - \alpha \tag{2.39}$$

In the paraxial approximation we can put

$$\phi = x/R \tag{2.40}$$

When maintaining the same sign convention as in Section 2.2, R will be negative, and so also the angle α' (α' is positive clockwise from the negative z-axis). Put into Equation (2.39), this gives

$$\alpha' = \alpha + 2\frac{x}{R} \tag{2.41}$$

The transformation at point A therefore can be written as

$$\begin{pmatrix} x' \\ \alpha' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2/R & 1 \end{pmatrix} \begin{pmatrix} x \\ \alpha \end{pmatrix} \tag{2.42}$$

Comparing this with the object–image transformation matrix, Equation (2.34), we get for the focal length of the spherical mirror

$$f = -\frac{R}{2} \tag{2.43}$$

Figure 2.7 shows four rays from an object point that can be used to find the location of the image point. Note that one of the rays goes through C and the image point. When approaching the mirror from beyond a distance $2f = R$, the image will gradually increase

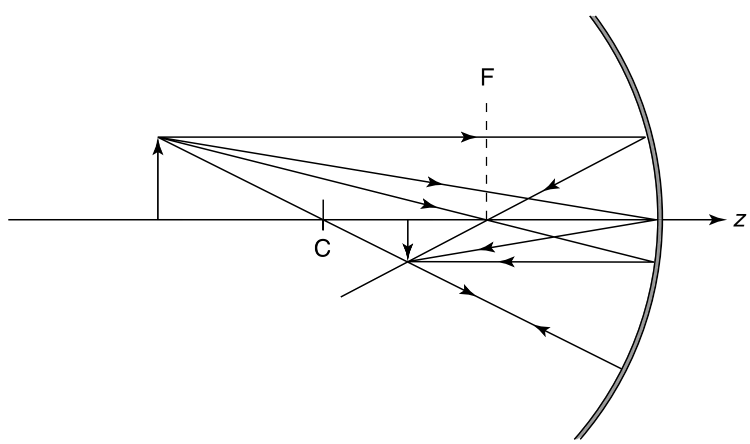


Figure 2.7 Imaging by a reflecting spherical surface

until at $2f$ it appears inverted and life-size. Moving still closer will cause the image to increase until it fills the entire mirror with an unrecognisable blur. Decreasing the distance further, the now erect, magnified image will decrease until the object rests on the mirror where the image is again life-size. The mirror in Figure 2.7 is concave. A mirror with opposite curvature is called convex. It is easily verified that a convex mirror forms a virtual image.

2.6 ASPHERIC LENSES

From school mathematics we learn that rays incident on a reflecting paraboloid parallel to its axis will be focused to a point on the axis. This comes from the mere definition of a parabola which is the locus of points at equal distance from a line and a point. The paraboloid and other non-spherical surfaces are called aspheric surfaces. The equation for the circular cross-section of a sphere is

$$x^2 + (z - R)^2 = R^2 \tag{2.44}$$

where the centre C is shifted from the origin by one radius R: see Figure 2.8. From this we can solve for z:

$$z = R \pm \sqrt{R^2 - x^2} \tag{2.45}$$

By choosing the minus sign, we concentrate on the left hemisphere, and by expanding z in a binomial series, we get

$$z = \frac{x^2}{2R} + \frac{1 \cdot x^4}{2^2 2! R^3} + \frac{1 \cdot 3 \cdot x^6}{2^3 \cdot 3! R^5} + \cdots \tag{2.46}$$

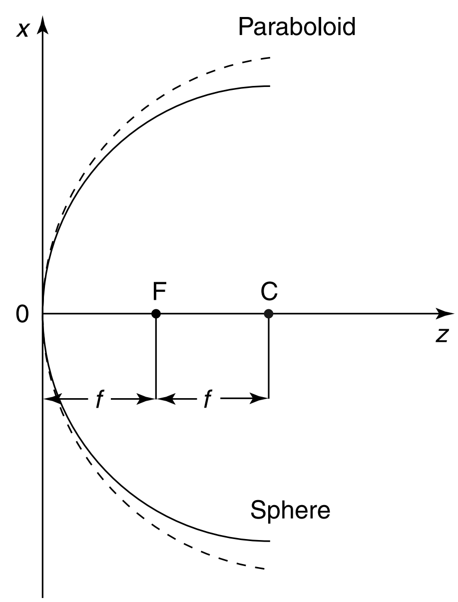


Figure 2.8

The equation for a parabola with its vertex at the origin and its focus a distance f to the right (see Figure 2.8) is

$$z = \frac{x^2}{4f} \tag{2.47}$$

By comparing these two formulas, we see that if $f = R/2$, the first contribution in the series can be thought of as being parabolic, while the remaining terms (in x^4 and higher) represent the deviation therefrom. Evidently this difference will only be appreciable when x is relatively large compared to R. In the paraxial region, i.e. in the immediate vicinity of the optical axis, these two configurations will be essentially indistinguishable. In practice, however, x will not be so limited and aberrations will appear. Moreover, aspherical surfaces produce perfect images only for pairs of axial points – they too will suffer from aberrations.

The best known aspherical element must be the antenna reflector for satellite TV reception. But the paraboloidal configuration ranges its present-day applications from flashlight and auto headlight reflectors to giant telescope antennas. There are several other aspherical mirrors of some interest, namely the ellipsoid and hyperboloid. So why are not aspheric lenses more commonly used? The first and most immediate answer is that, as we have seen, in the paraxial region there is no difference between a spherical and a paraboloidal surface. Secondly, paraboloidal glass surfaces are difficult to fabricate. We also might quote from Laikin (1991): 'The author's best advice concerning aspherics is that unless you have to, don't be tempted to use an aspheric surface'. An important exception is the video disk lens. Such lenses are small with high numerical aperture operating at a single laser wavelength; they cover a very small field and are diffraction limited. A recent trend in the manufacture of these lenses is to injection-mould them in plastic. This has the advantage of light weight and low cost (because of the large production volume) and an aspheric surface may be used.

2.7 STOPS AND APERTURES

Stops and apertures play an important role in lens systems.

The aperture stop is defined to be the aperture which physically limits the solid angle of rays passing through the system from an on-axis object point. A simple example is shown in Figure 2.9(a) where the hole in the screen limits the solid angle of rays from the object at P_o. The rays are cut off at A and B. The images of A and B are A' and B'. To an observer looking back through the lens from a position near P'_o it will appear as if A' and B' are cutting off the rays. If we move the screen to the left of F, we have the situation shown in Figure 2.9(b). The screen is still the aperture stop, but the images A', B' of A and B are now to the right of P'_o. To an observer who moves sufficiently far to the right it still appears as if the rays are being cut off by A' and B'.

A 'space' may be defined that contains all physical objects to the right of the lens plus all points conjugate to physical objects that are to the left of the lens. It is called the image space. In Figures 2.9(a, b) all primed points are in image space. The image of the aperture stop in image space is called the exit pupil. To an observer in image space it appears either as if the rays converging to an on-axis image P'_o are limited in solid angle

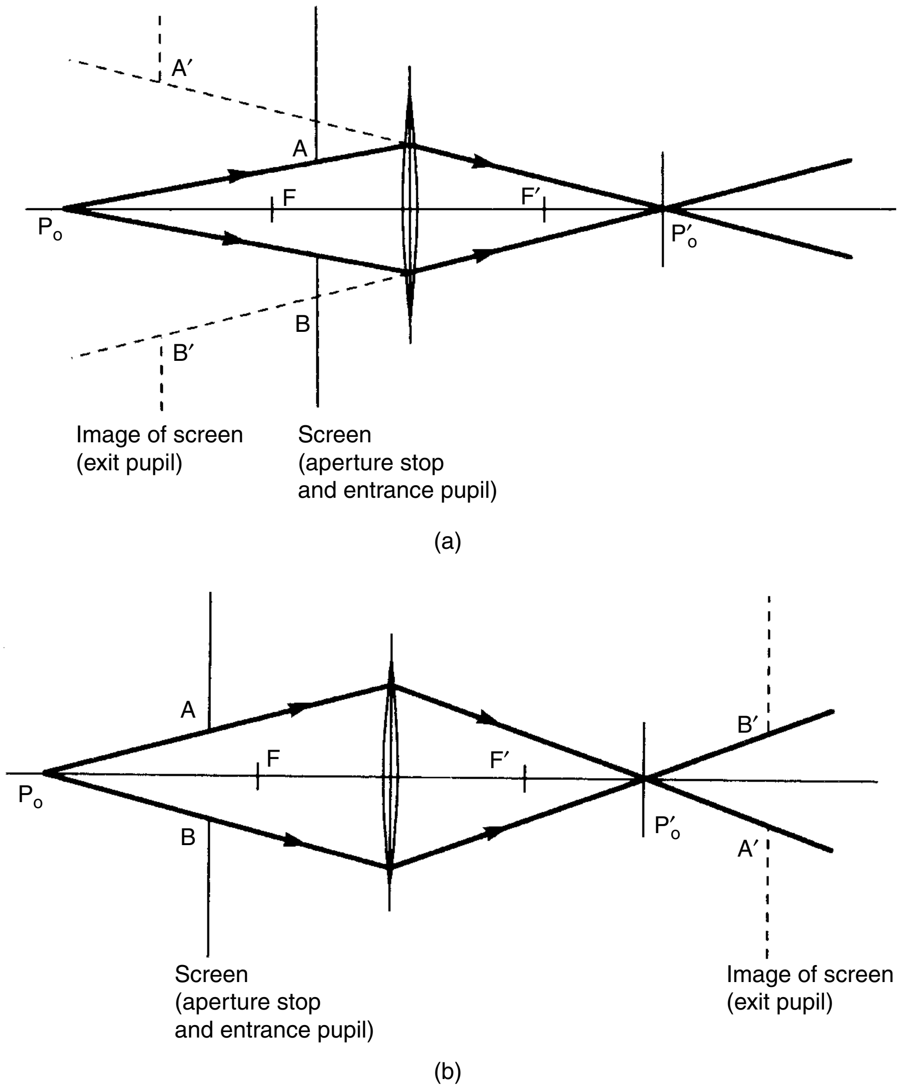


Figure 2.9 Illustrations of entrance and exit pupils

by the exit pupil A′B′ as in Figure 2.9(a) or as if the rays diverging from P'_o are limited in solid angle by A′B′ as in Figure 2.9(b).

By analogy to the image space, a space called the object space may be defined that contains all physical objects to the left of the lens plus all points conjugate to any physical object that may be to the right of the lens. In Figure 2.9(a, b) all unprimed objects are in the object space. The image of the aperture stop in the object space is defined as the entrance pupil. The aperture stop in Figure 2.9(a, b) is already in the object space, hence it is itself the entrance pupil.

In a multilens system some physical objects will be neither in the object nor in the image space but in between the elements. If a given point is imaged by all lens elements to its right, it will give an image in the image space; if imaged by all elements to its left, it will give an image in the object space. A systematic method of finding the entrance pupil is to image all stops and lens rims to the left through all intervening refracting elements of the system into the object space and find the solid angle subtended by each at P_o. The one with the smallest solid angle is the entrance pupil, and the physical object corresponding to it is the aperture stop. Alternatively we may image all stops and lens rims to the right through all intervening refractive elements into the image space and determine the solid angle subtended by each image at P'_o. The one with the smallest solid angle is the exit pupil, and the corresponding real physical object is the aperture stop.

2.8 LENS ABERRATIONS. COMPUTER LENS DESIGN

The ray-tracing equations used in the theory of Gaussian optics are correct to first order in the inclination angles of the rays and the normals to refracting or reflecting surfaces. When higher-order approximations are used for the trigonometric functions of the angles, departures from the predictions of Gaussian optics will be found. No longer will it be generally true that all the rays leaving a point object will exactly meet to form a point image or that the magnification in a given transverse plane is constant. Such deviations from ideal Gaussian behaviour are known as lens aberrations. In addition, the properties of a lens system may be wavelength- dependent, known as chromatic aberrations.

Monochromatic aberrations may be treated mathematically in lowest order by carrying out the ray-tracing calculations to third order in the angles. The resulting 'third-order theory' is itself valid only for small angles and for many real systems calculations must be carried out to still higher order, say fifth or seventh. (For a centred system with rotational symmetry, only odd powers of the angles will appear in the ray-tracing formulas.)

Most compound lens systems contain enough degrees of freedom in their design to compensate for aberrations predicted by the third-order theory. For real systems the residual higher-order aberrations would still be present, and there are not enough design parameters to eliminate all of them as well. The performance of a lens system must be judged according to the intended use. The criteria for a telescope objective and for a camera lens for close-ups are quite different.

Third-order monochromatic aberrations can be divided into two subgroups. Those belonging to the first are called spherical aberrations, coma and astigmatism and will deteriorate the image, making it unclear. The second type cover field curvature and distortion, which deform the image. Here we will not treat lens aberrations in any detail. Figure 2.10 illustrates spherical aberration, and in Section 10.4.1 distortion is treated in

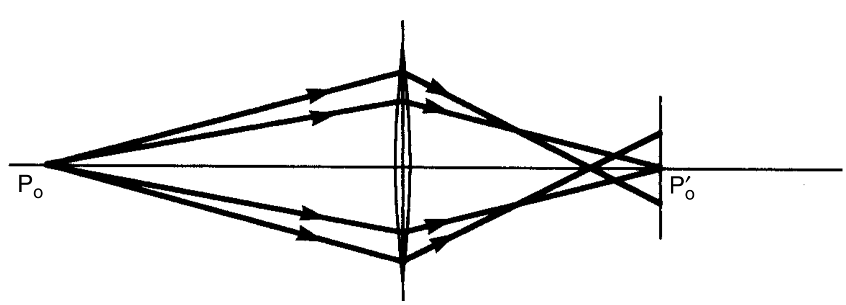


Figure 2.10 Spherical aberration. The focus of the paraxial rays is at P'_o. The marginal rays focus at a point closer to the lens

some detail. Because of the complexity of the higher-order aberrations they are usually treated numerically. Now lens design computer programs are available commercially. Such programs trace a lot of different rays through the system and the points where they intersect the image plane is called a spot diagram. By changing the design parameters, the change in the spot diagram can be observed. Some computer programs do such analyses automatically. The computer is given a quality factor (or merit function) of some sort, which means how much of each aberration is tolerated. Then a roughly designed system which, in the first approximation, meets the particular requirements is given as input. The computer will then trace several rays through the system and evaluate the image errors. After perhaps twenty or more iterations, it will have changed the initial configuration so that it now meets the specified limits on aberrations. However, a quality factor is somewhat like a crater-pocked surface in a multidimensional space. The computer will carry the design from one hole to the next until it finds one deep enough to meet the specifications. There is no way to tell if that solution corresponds to the deepest hole without sending the computer out again and again meandering along totally different routes.

2.9 IMAGING AND THE LENS FORMULA

Before studying specific lens systems, let us have a closer look at the imaging process and the lens formula. We have found that a general imaging system is characterized by the focal length f and the positions of the two principal planes H and H′ which determine the four cardinal points F, F′, H and H′: see Figure 2.11. Imaging takes place between conjugate planes in object and image space, and the object and image planes are related by the lens formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{2.48}$$

where a and b are measured from the principal planes. Note that both a and b can assume values between $-\infty$ and ∞. If the object plane lies to the right of the vertex of the first refracting surface, we have no real object point, but rays that converge to a virtual object point behind the first refracting surface: see Figure 2.12(c). In the same way we have a virtual image plane if the image lies to the left of the last vertex of the lens system: Figure 2.12(b). The rays diverge as if coming from this virtual image point, but they do not intersect there. Only if rays really intersect at the image point do we have a real image point and that happens only if the image plane lies to the right of the last vertex

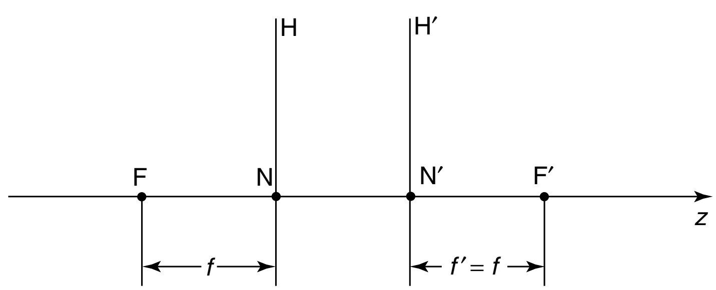


Figure 2.11 Principal points

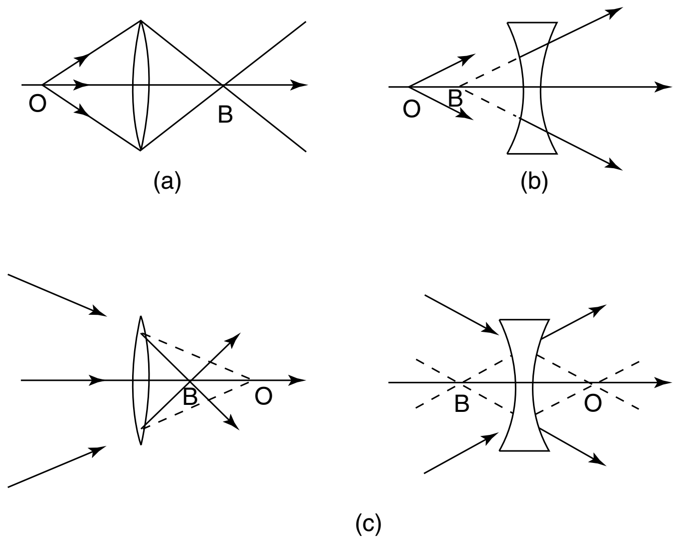


Figure 2.12 Real and virtual object (O) and image (I) points: (a) real object, real image; (b) real object, virtual image; (c) virtual object, real image; and (d) virtual object, virtual image

of the system. The focal length can also assume values in the range $[-\infty, \infty]$. When $f > 0$, we have a positive (or collecting) lens, and when $f < 0$ we have a negative lens: see Figure 2.12(b). For a negative lens, F is to the right of H, while F′ is to the left of H′.

In addition to the above-mentioned cardinal points, we also have the so-called nodal points N and N′ on the axis: see Figure 2.11. A ray incident on N in the object space leaves N′ in the image space in the same direction. Rays through nodal points therefore are parallel, which means that the angular magnification between N and N′ is unity. With the same refractive index in front and behind the lens ($n = n'$), we get $m_x m_\alpha = 1$, which means that the nodal points must lie in the principal planes. With unequal indices, the nodal points move away from the principal planes.

2.10 STANDARD OPTICAL SYSTEMS

It should be remembered that the systems described below are visual instruments of which the eye of the observer is an integral part.

2.10.1 Afocal Systems. The Telescope

An afocal system has zero power P. This can be realized by two lenses separated by a distance t equal to the sum of the individual focal lengths, $t = f_1 + f_2$: see Figure 2.13. The system matrix becomes

$$M = \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & (f_1 + f_2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} = \begin{bmatrix} -f_2/f_1 & (f_1 + f_2) \\ 0 & -f_1/f_2 \end{bmatrix} \quad (2.49)$$

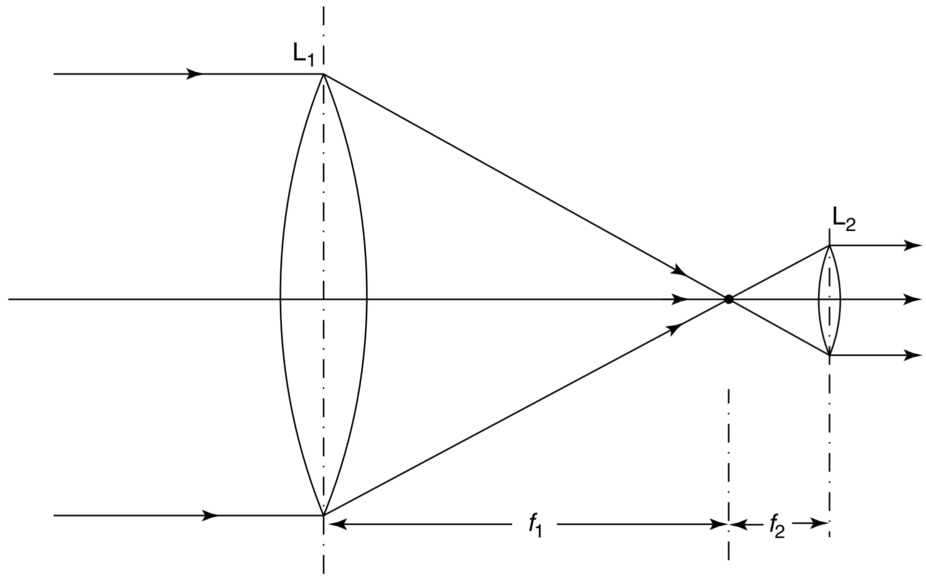


Figure 2.13 The telescope

We see that the M_{21}-element is zero, which means $P = 0$. Computing the transformation from a plane a distance d in front of the first lens to a plane a distance d' behind the second lens gives

$$M_{dd'} = \begin{bmatrix} 1 & d' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -f_2/f_1 & (f_1 + f_2) \\ 0 & -f_1/f_2 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -f_2/f_1 & [(f_1 + f_2) - f_2 d/f_1 - f_1 d'/f_2] \\ 0 & -f_1/f_2 \end{bmatrix} \tag{2.50}$$

Assuming d and d' to be the object and image planes, the (1,2)-element must be zero, and we get

$$M_{dd'} = \begin{bmatrix} -f_2/f_1 & 0 \\ 0 & -f_1/f_2 \end{bmatrix} = \begin{bmatrix} m_x & 0 \\ 0 & m_\alpha \end{bmatrix} \tag{2.51}$$

Contrary to other lens systems, the lateral magnification

$$m_x = -f_2/f_1 \tag{2.52}$$

is constant and independent of the object and image distances. This implies that an afocal system does not have principal planes with mutual unit magnification. The object–image relation is also very different from the usual lens formula:

$$(f_2/f_1)d + (f_1/f_2)d' = f_1 + f_2 \tag{2.53a}$$

or

$$d' = (f_1 + f_2)(f_1/f_1) - (f_2/f_1)^2 d \tag{2.53b}$$

A telescope is an afocal system with $f_1 > |f_2|$ giving $|m_x| < 1$. The reason for this seeming paradox is that when $d \to \infty$ it is the angular magnification $m_\alpha = -f_1/f_2$ that determines how large the image looks. The virtual image is demagnified by a factor $m_x = -f_2/f_1$, but this is contrasted by being focused at a distance $d' \approx dm_\alpha^2$ and is moved closer by a factor $m_x^2 = (f_2/f_1)^2$. The angular magnification then becomes $m_x/m_x^2 = 1/m_x = -(f_1/f_2) > 1$.

Since negative lenses have virtual focal points and the focal points in an afocal system must coincide, the lens with the longest focal length must always be positive. The lens with the shortest focal length can be either negative, giving an erect image with $m_x > 0$ (Galileo's telescope, the theatre telescope), or positive, giving an inverted image. In binoculars, the image is erected by inverting the image in two total reflecting prisms. It should be noted that when observing faint stellar objects, large angular magnification is not sufficient if the irradiance is too low. The light-collecting capacity is determined by the front lens. Therefore, when judging the quality of a stellar telescope, the diameter of the front lens is a more important parameter than the magnification. However, large-aperture lenses inevitably give more aberrations. Since large-aperture corrected mirrors are easier to fabricate than lenses, stellar telescopes are often equipped with mirrors as front objectives. Figure 2.14 shows some of the most common designs.

2.10.2 The Simple Magnifier

The unaided eye focuses on an object when the object distance is larger than about $d_o = 25$ cm. The angular resolution (determined by the rods and cones) is about $0.5' = 0.5/60 = 1/120$ deg $= 1/7000$ radian. At a distance of 25 cm we therefore cannot distinguish object details less than 0.07 mm. To observe smaller objects we can use a magnifier.

In Figure 2.15 the object of height h is placed at a distance $a < f$, where f is the focal length of the magnifier. The resulting virtual image is located a distance b in front of the lens, given by the lens formula $1/a + 1/b = 1/f$. Since d_o is the closest distance the eye can focus, we put $b = -d_o$ (b is negative), giving

$$a = \frac{d_o f}{d_o + f} \tag{2.54}$$

and the magnification

$$m = \frac{d_o}{a} = \frac{d_o + f}{f} = \frac{d_o}{f} + 1 \tag{2.55}$$

For a magnifier with $f = 5$ cm, the effective magnification is about 5–6 depending on how the observer focuses.

A simple uncorrected magnifier has rather poor imaging qualities. Similar but well-corrected systems are applied as oculars in visual instruments. An ocular is a well-corrected magnifier for visual observation of intermediate images in optical systems. Since an intermediate image can be virtual, negative lenses can also be used as oculars.

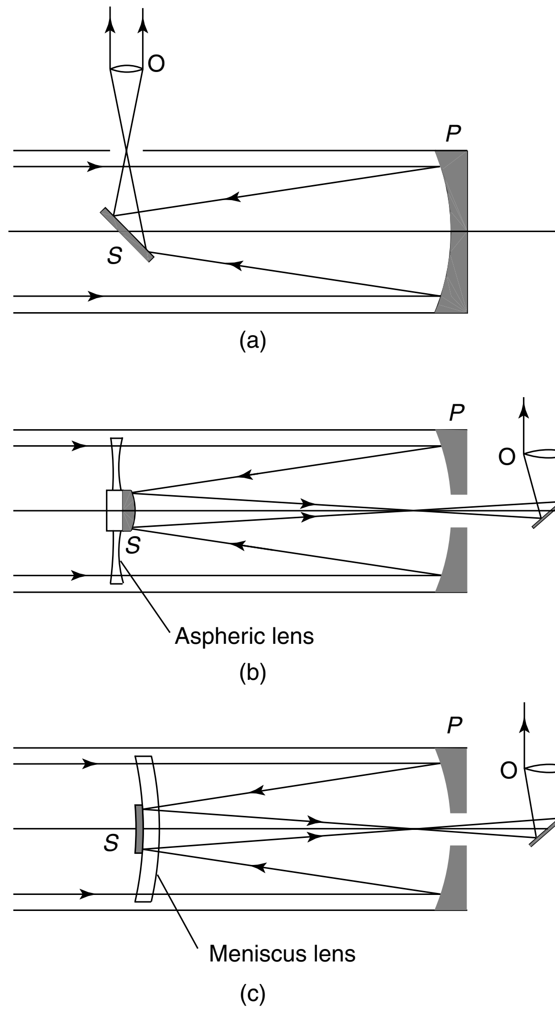


Figure 2.14 Some common telescope designs: (a) Newtonian; (b) Schmidt–Cassegrain; and (c) Maksutov–Cassegrain. P = primary mirror, S = secondary mirror, O = ocular

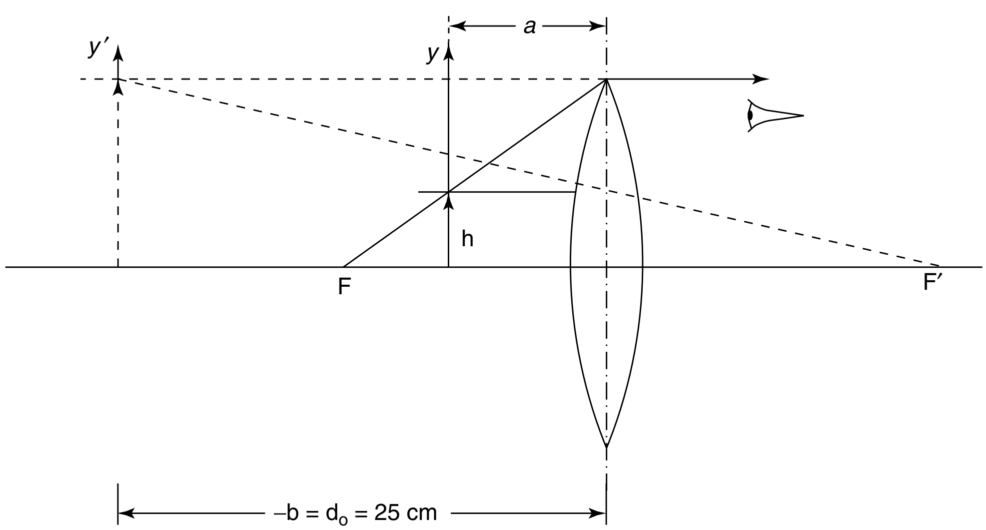


Figure 2.15 The simple magnifier

2.10.3 The Microscope

A microscope is used for observation of very small objects where the magnification of the viewing angle is so large that the assumptions of paraxial optics are no longer valid. The magnification can be several hundreds, the focal length lies in the millimetre range and the objective lens is composed of several elements (compound lens). Microscopes are specialized and standardized instruments consisting of exchangeable objectives with various focal lengths f_{ob}, but which focus an intermediate image at a fixed distance $b = T = 16$ cm (the tubus length). The magnification of the objective is therefore given by $m_{\text{ob}} \approx T/f_{\text{ob}}$. For a 40× objective we therefore get $f_{\text{ob}} \approx 16$ cm/40 = 4 mm. The magnified intermediate image is observed by the ocular, which focuses at infinity, giving a magnification of the viewing angle equal to $d_{\text{o}}/f_{\text{oc}} \approx 25\ \text{cm}/f_{\text{oc}}$. A 10× ocular therefore has a focal length f_{oc} equal to 2.5 cm. The overall magnification becomes $m_{\text{ob}} \cdot m_{\text{oc}} \approx T d_{\text{o}}/(f_{\text{ob}} f_{\text{oc}})$, which in our example gives $40 \times 10 = 400$.

PROBLEMS

2.1 Verify directly by matrix methods that use of the matrix in Equation (2.34) will yield values of (x', α') for rays 1, 2, 3, 4 in Figure 2.5 so that they behave as shown.

2.2 Consider the system shown in Figure P2.1 where the focal lengths of the first system are f_1, f_1' and those of the second f_2, f_2'. The respective powers are

$$P_1 = \frac{n_1}{f_1} = \frac{n_1'}{f_1'}$$

$$P_2 = \frac{n_2}{f_2} = \frac{n_2'}{f_2'}$$

(a) Find the transformation matrix $M_{\text{H}_1\text{H}_2'}$ between the first principal plane of the first system and the second principal plane of the second system.
We denote the principal planes of the whole system by H and H′ and the distances $\text{HH}_1 = D$ and $\text{H}_2'H' = D'$.

(b) Express the transformation matrix $M_{\text{HH}'}$ between H and H′ in terms of the total power P and n and m'.

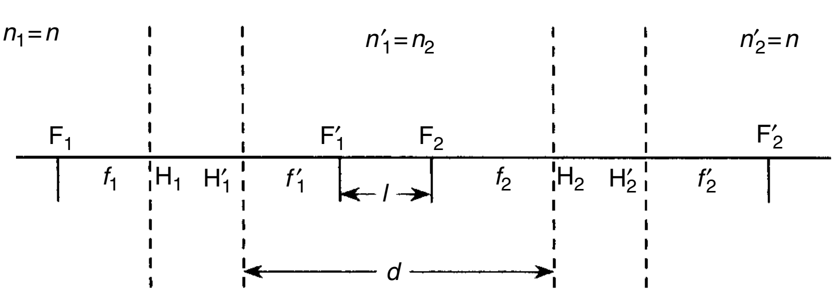


Figure P2.1

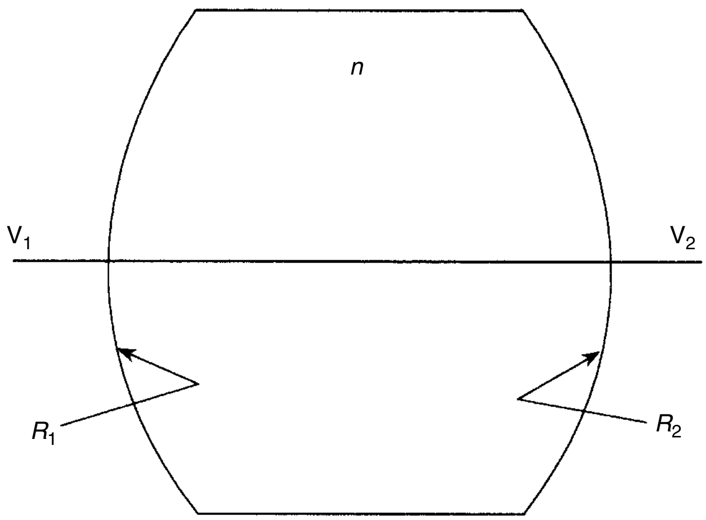


Figure P2.2

(c) Find the total power of the system.

(d) Find D and D'.

2.3 A doublet consists of two lenses with principal plane separation $d = f_1' + f_2 + l$, see Figure P2.1. We set $n_2 = n_2' = 1$.

(a) Find the power P of the doublet in terms of P_1, P_2 and l.

(b) Find the first and second focal lengths.

2.4 Find the power and the locations of the principal planes for a combination of two thin lenses each with the same focal length $f > 0$ separated by a distance d: (a) where $d = f$, (b) where $d = 3f/4$.

2.5 Show that the combination of two lenses having equal and opposite powers a finite, positive, distance d apart has a net positive power P, and find P as a function of d.

2.6 A thick lens as shown in Figure P2.2, is used in air. The first and second radii of curvature are $R_1 > 0$ and $R_2 < 0$, the index is $n > 1$, and the thickness $|V_1V_2|$ is d. What will be the aperture stop for this lens for an axial object at a general distance S_1 to the left of V_1? Is the aperture stop always the same? (No calculation is necessary to solve this problem.)

2.7 A thin lens L_1 with a 5.0 cm diameter aperture and focal length +4.0 cm is placed 4.0 cm to the left of another lens L_2 4.0 cm in diameter with a focal length of +10.0 cm. A 2.0 cm high object is located with its centre on the axis 5 cm in front of L_1. There is a 3.0 cm diameter stop centered halfway between L_1 and L_2. Find the position and size of (a) the entrance pupil, (b) the exit pupil, (c) the image. Make a brief sketch to scale.